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Game-theoretic model of asymmetrical multipath selection in pervasive computing environment

Jingyu Wang [a,*], Jianxin Liao [a], Tonghong Li [b], Jing Wang [a]

[a] State Key Laboratory of Networking and Switching Technology, Beijing University of Posts and Telecommunications, Beijing, 100876, PR China
[b] Technical University of Madrid, Madrid, 28660, Spain

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ABSTRACT

In order to transfer the ever increasing multimedia data we need to make use of multiple paths to realize the bandwidth aggregating. In pervasive computing environment, the combination of ubiquitous overlay networks and the capacity of multihoming can provide the possibility of exploiting plentiful available paths. Thus, we introduce a Pervasive Multipath Architecture (PEMA), in which we use the game theory to investigate the selfish strategic collaboration of multiple heterogeneous overlays when they are allowed to use the massively-multipath transfer. Overlay networks are modeled as “players” in this multipath selection game, and we study the asymmetric case where all overlays have different Round Trip Times (RTT) and different degrees of waste. We demonstrate the existence and uniqueness of Nash equilibrium (NE). In addition, we find that overlays differing only in their *RTTs* still receive proportional throughput shares and utilities at the NE. However, if overlays differ only in their degrees of waste, a more wasteful overlay has a larger utility and a larger throughput (bandwidth) share than a less wasteful overlay. Since maintaining connected paths consumes resource, we further consider a more generalized cost function where an overlay's total resource consumption includes both transferring data packets and maintaining path connections. In this general game, we find that the overlay is more conservative, which opens smaller number of paths and obtains smaller efficiency loss at the NE than in other simplified games. Our simulations confirm the effectiveness and friendliness of multipath transfer for a range of path numbers and in the presence of multi-overlay traffic.

1. Introduction

By using the ubiquitous and overlapping access networks, our world has been interconnected as a pervasive computing environment consisting of massive intelligent network devices, which can process and transport information to adapt to the associated context and activity. It is challenging to enable multiple access networks to cooperate autonomously to realize the ubiquitous access and multipath transfer between any two end hosts. In this environment, the multipath transfer technologies [1,2] have gained more and more attention from academy and industry, which could maximize the use of network resources and multiply the available bandwidth. On the other hand, traditional routing policies are restrictive, which limits the communication between source–destination pairs to only one path, although there often exist more than

* Corresponding author.
E-mail addresses: wangjingyu@bupt.edu.cn (J. Wang), liaojx@bupt.edu.cn (J. Liao), tonghong@fi.upm.es (T. Li), wangjing@bupt.edu.cn (J. Wang).

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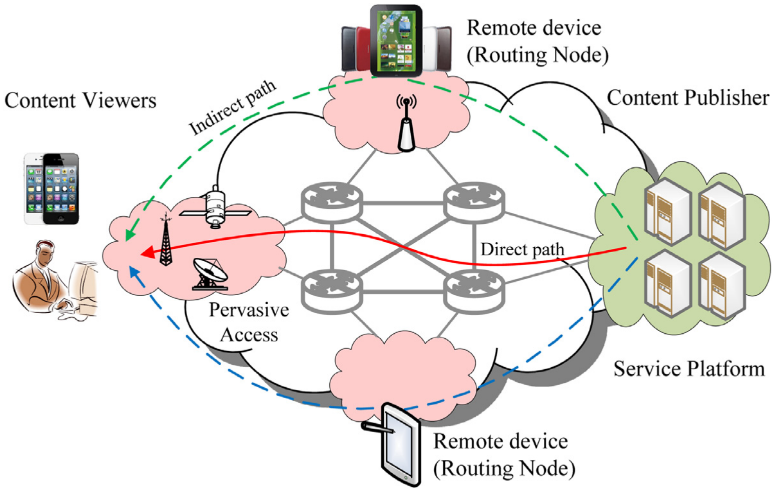

Fig. 1. A multi-homing and overlay network with multiple path support.

one indirect paths through the overlay routing via overlay peer(s). Earlier studies [3,4] have demonstrated the benefits of overlay multipath transfer to achieve high available bandwidth, good loss patterns and bounded delay in the best-effort Internet environment. As the last mile bandwidth has been set to increase dramatically over the last few years, as shown in Fig. 1, we expect that bottlenecks will be shifted away from the network edges and the end-to-end data transfers will be constrained by the capacity of core links. Thus, we will have the opportunity to exploit path diversity and use multiple routing paths concurrently to fully saturate the available core link bandwidth for high-volume data transfers, e.g. video streaming or inter-datacenter bulk transfers.

In this system, end users are effectively provided with a large set of potential paths from which they choose a small subset of paths to improve the performance. As the selected paths may negatively affect each other and the underlying network, there is a need for the management system that controls and adapts their behaviors to meet their specific demands and those of the network and service providers. The following questions arise naturally: How many paths are required? How can we design a multipath selection mechanism that only opens the necessary and sufficient number of paths to exploit the multipath transfer capability? One of the main objectives of the multipath selection is to establish a number of rules guaranteeing that common resources are fairly shared among all the overlays. In most prior models, it has been assumed that each flow is assigned a single path between its source and destination. However, in multipath models each flow can be split into multiple paths simultaneously by a unified control. As using excessive paths can degrade the overall system performance, we should select the number of paths as small as possible. Selfish overlay networks are often accused to be unfriendly when they share network resources with traditional or other overlay flows. As the paths are shared between many flows, the greedy selection behavior can easily lead to unstable network state. The cooperation is hard to maintain due to the characteristics of selfish overlays. Even if Nash equilibrium (NE) appears, it may not be optimal. The number of paths used by each application is determined by the multipath selection according to the observed qualities of total networks.

In this paper we analyze a scenario where multiple selfish overlays deliver their traffic in a shared multipath network infrastructure between the common source and destination. By using a game-theoretic framework, we study the impact of selfish behaviors of overlays when multipath transfer is allowed. One of our motivations comes from the observation that more and more users use Peer-to-peer (P2P) applications (e.g., PPLive, BitTorrent) to construct the overlay network to realize the efficient multipath transfer. For example, PPLive maintains 4 active paths and an additional path periodically chosen at random according to the throughput measured. In our scenario, a number of overlays compete for the capacity of a single bottleneck link. Overlays are modeled as “players” in the game, where each player has infinite amounts of data to send and is allowed to open a number of concurrent paths. The criterion of optimality is trying to maximize each player’s utility function, which is a combination of a payoff and a cost.

We assume that the payoff received by a player is a continuous, concave and increasing function of its received data throughput. Regarding the cost, the player is concerned with the number of its paths. This cost accounts for the cost of maintaining the paths opened, which is specific to the overlay. In this game, a pure-strategy Nash equilibrium [5] is a vector of the numbers of paths of all overlays, and each player cannot benefit from unilaterally deviating from this NE.

We investigate the effect of the number of TCP paths on the network performance with a focus on a game-theoretic study of selfish player behaviors. We assume that each player uses “standard TCP” and is allowed to choose the number of concurrent paths. Our result for multipath networks presented in this paper is consistent with the following conclusion drawn in the traditional unipath networks [6]: when the users use TCP New Reno loss-recovery [7] in combination with drop-tail queue management, the equilibrium strategies of the users are quite efficient for fair resource allocation. Although playing NE is fair, it does not guarantee that each player be given the best possible reward. When the players are non-cooperative, playing Nash is the best-response strategy to be applied. However, if the players are cooperative, each of them may obtain a better reward.

The contributions of this paper are summarized as follows.

(1) We introduce a Pervasive Multipath Architecture (PEMA), which combines the benefits of multihoming and overlay routing. This overlay network architecture can be applied to build a multipath transport through the cooperation of other relaying devices between the source and destination. Multihoming enables the availability of more heterogeneous access links to enhance the diversity of end-to-end paths, and thus the user perceived quality of experience for the real-time multimedia service can be guaranteed.
(2) To determine the path number selected in PEMA, we formulate the problem of path number selection as the multipath selection game, and show the existence of an NE in the multipath selection game. The players of the game are the PEMA overlays and the action of the game is that each source node selects the proper number of paths to forward its flows, trying to reach its own best performance. We discuss the asymmetric case where all overlays have different Round Trip Times (RTT) and different wastefulness levels, and demonstrate the existence and the uniqueness of an NE.
(3) We find that overlays differing only in their *RTT*s still receive proportional throughput shares and utilities at the NE. However, if overlays differ only in their wastefulness levels, a more wasteful overlay has a larger utility and a higher throughput (bandwidth) share than a less wasteful overlay. Since the NE in the continuous game can take non-integer values, we further investigate an integer variant where an overlay is only allowed to open an integer number of paths. We observe that a pure-strategy integer NE always exists in this integer game.
(4) We propose a generalized cost function where an overlay's cost includes transferring data packets and maintaining the connected paths. In this model, the interaction between the overlays is defined as "general game", and the previous simple model is called as "special game" to differentiate them. We observe that overlays in "general game" are more conservative. Hence, the overlay opens a smaller number of paths at the NE in "general game" than in "special game". Even though the upper bound of "general symmetric game" is the same as that of "special game", its actual efficiency loss is smaller than that of "special game".

The rest of the paper is organized as follows. Section 2 summarizes the related work. In Section 3, we introduce the pervasive multipath architecture. Formulation of multipath selection game and network optimization problem is presented in Section 4. In Section 5, we study the continuous and integer multipath selection respectively for the asymmetric game. Then a general case is discussed in Section 6. We present simulation and evaluation results in Section 7. Section 8 concludes the paper and presents the ideas for future work.

2. Related work

Nowadays Game theory is widely used as a tool for studying, modeling and analyzing the interactions between individuals strategically. It has been used to study various communication and networking problems including routing, service provisioning, access control and flow-rate controlling by formulating them as either cooperative [8] or non-cooperative games [9]. An overview of using game theory to solve the network selection problem is provided in [10]. The book [11] presents a collection of fundamental issues and solutions in applying game theory in different wireless communications and networking domains. When users do not cooperate and do not respect the protocol rules, unfair or unstable behaviors may emerge in the system. This problem of the TCP protocol has already been addressed by using a game-theoretic perspective. For example, Akella et al. [6] gave an excellent analysis of TCP behavior in the context of Game Theory, which attempts to characterize the performance of TCP in the presence of selfish users.

Another interesting related congestion game is given in [12], where the authors explore the interaction between overlay networks and traffic engineering. They use a game-theoretic framework in which infinitesimal users of a network select the source of content, and the Traffic Engineer (TE) decides how the traffic will be routed through the network. They also formulate a game and prove the existence of equilibria. In [13], the authors use best-reply dynamics to model the non-cooperative interactions between the P2P overlay and the network-level TE behaviors. The interaction between selfish overlays is also studied in several works such as [14,15], where multiple overlays compete for limited network resources. To address the selfish behavior of the overlays, most of research [16] seeks to maximize their utility by taking into account the actions of each other. Also some researches [17] do not take into account the actions of each other when studying the individuals. For above both cases, Zhang et al. [18] prove the existence of Nash equilibrium among self-interested unstructured P2P file sharing applications. A routing scheme [19] is designed to dynamically select the best available forwarding paths by using a game-theoretic approach over a multipath routing protocol. Selecting the best radio access technology (RAT) [20] is formulated as a non-cooperative game, where its convergence, efficiency and practicality are studied. In our earlier works [21], we propose an open multiplane framework for Next Generation Service Overlay Network, in which different functional overlays can systematically be coordinated with each other.

There are also attempts to design optimal traffic controllers to optimally spread data over multiple overlay paths. With the objective of achieving the minimum packet delay, the optimal multipath data transfer problem was studied in [22]. On the other hand, aiming at maximizing an aggregate sending rate of a source over multiple overlay paths, the authors of [23] develop a multipath rate controller that randomly chooses a transmission path by following a throughput proportional selection scheme. Although different approaches have been used to address various multipath data transfer problems, none of them considered the problem from the multiple controller's perspective. Therefore, these approaches may be not suitable for the recent advances of Internet Engineering Task Force (IETF) working group in multipath transfer, such as CMT-SCTP [24,1] and Multipath TCP [2], where multiple congestion controllers compete for the same shared available bandwidths. In [25], the authors extend the notion of "fairness" from single-path transport to multipath transport, and introduce the relevant congestion control approaches.

In our earlier works, we propose cmpSCTP [26], in which flow control and congestion control mechanism are separated and congestion control is performed for each path independently. We also introduce a new problem of selecting the limited numbers of selfish paths [27] to realize the effect of path diversity in concurrent multipath transfers, but the impact and fairness of other paths are ignored. A cooperative overlay optimization approach [28] is proposed to improve the performance of the whole network, while considering the fairness between co-existing overlays, in which overlays can form coalitions freely for the purpose of cooperation. For the collaborations of multipath selection, we discuss the symmetric case [29] where all overlays have the same *RTT* and wastefulness level. In reality, overlays may not have the same utility function. In this paper, we extend the symmetric game to the asymmetric game. For the asymmetric game, we also prove the existence and stability of an NE when the path number is real or integer.

3. Pervasive Multipath Architecture (PEMA)

The goal of pervasive computing is to create ambient intelligence where network devices provide connectivity and services all the time. Nowadays, the end-user mobile devices are usually multihomed, having several heterogeneous network interfaces allows a concurrent-connected approach to networking. In this section, we discuss the architecture of our PEMA and demonstrate a typical instance of video delivery under this architecture.

3.1. Overlay multipath based on multihoming access

In this pervasive computing environment, we consider: (1) the multihoming ability of the mobile devices to exploit heterogeneous network technologies and to be connected to different access points simultaneously; and (2) the cooperation of a large number of relaying mobile devices providing multiple available routes for a mobile device. Let us consider a scenario where the streaming content needs to be delivered for mobile users in resource-limited networks, in which multihoming access and overlay multipath routing are used. Packet transmission is always sent to/from the central server, without any pure peer-to-peer connection between any two arbitrary mobile nodes. The network construction uses a lightweight unstructured mechanism, which does not impose a rigid relation between the overlay topology and the place where resources or their indices are stored. In this manner, overlay networks are easier to implement and reconfigure promptly even in dynamic environments. A mobile terminal can connect to several access networks simultaneously and has multiple IP addresses. Relaying devices communicate among one another to provide multiple available paths for a mobile device due to the rich connectivity. Overlay routing can leverage the existence of multiple, simultaneous paths in backbone networks. Obviously, multihoming access and overlay networks can provide significant availability gains, that is, the device can receive data through more than one (overlay) paths.

The video server can stream the data to the client simultaneously on several disjoint paths, which are the entire end-to-end delivery path from the source to the destination. Some are direct paths, while the others traverse across cooperative overlay routers/relays and utilize the capacity of their transfer links. These overlay routers are terminal devices which have the corresponding forward function in the application-layer. In fact, a device may function as source/aggregator/helper and support one or more sessions for one or more services.

Based on the concept illustrated in Fig. 1, a pervasive multipath network can be formed, in which plentiful relaying devices cooperate to provide multiple paths for a mobile device. The overlay path between the source and destination may contain several overlay links, which are IP layer paths between the selected mobile devices. No matter the overlay node is the cooperative device or the application-layer router, it can route packets on dynamically pre-specified paths. Hence, we also consider overlay routing in network side where some legacy nodes are replaced with overlay nodes. Conceptually, we model the controllable node as the one operating in a network overlay on top of a legacy network. Due to the rich connectivity, PEMA can provide multiple paths for a mobile device to improve the network resources utilization.

3.2. Overlay routing layer of PEMA

These overlay nodes cooperate with each other to provide an overlay service platform [30], on top of which a variety of application-specific overlays can be constructed, such as overlay multicast, and Peer-to-Peer file sharing, etc. As overlay nodes are connected via IP-layer paths (overlay links), theoretically, there is an overlay link connecting each pair of overlay nodes.

The data plane of the multipath network introduces an overlay routing layer between the application and the underlying transport protocol. The function of the overlay routing layer is to route the traffic from the sender to the receiver over overlay network relays. An illustration of the pervasive multipath network data plane based on Wifi/4G access is shown in Fig. 2. Network overlays are used to realize new communication architectures in legacy networks. To accomplish this, messages

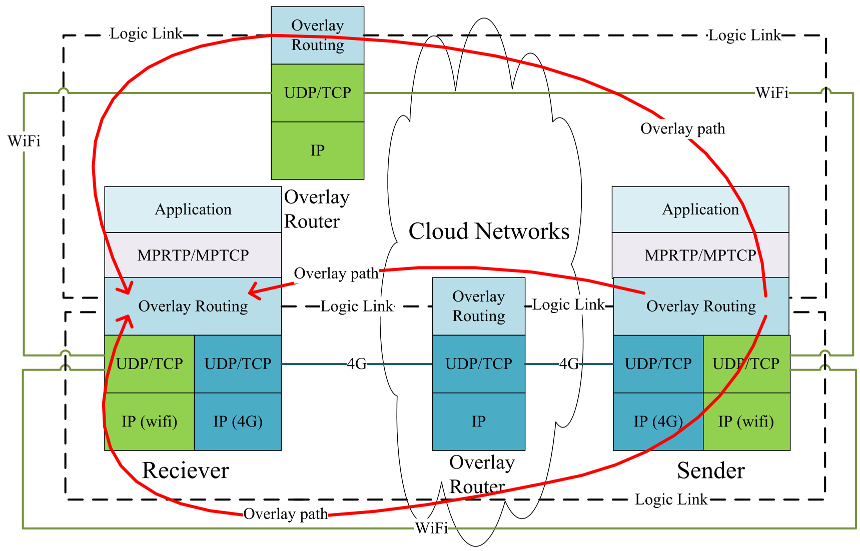

Fig. 2. Pervasive multipath network data plane.

from the new technology are encapsulated in the legacy format. Strategies for overlay routing describe the process of path computation. The overlay routing nodes are used as a distributed database by the content looking up service, and are also used to relay the application data. A sender host sends data packets to a receiver host via a sequence of reachable overlay routing nodes, by specifying the receiver's identifier instead of the receiver's locator (i.e., IP address). The overlay routing node will finally forward the data to the IP address correspondent to the specified identifier.

The overlay routing layer employs a source routing architecture similar to DSR (Dynamic Source Routing) [31]. The source routing architecture can reduce the complex routing processing and enable differentiating the per-hop behaviors for different streaming sessions. Each packet is specified the route when traveling from one overlay device to another. The exact path that a packet should take, from one overlay router to the next, is specified before the packet leaves the source. The routing decisions are handled by the source node, which uses source routing protocols to process the routing information in order to discover the optimal paths to a destination. The overlay routing mechanism is on demand based, that is, it does not require any kind of periodical message to be sent. So it eliminates the need for the periodic route advertisement and neighbor detection packets.

The control plane of the multipath network is used to establish, replace and release a data plane path between a sender and a receiver. The pervasive multipath network uses TCP for transporting overlay network control plane messages. Source route discovery is initiated by a source whenever a source has a data packet to send, but does not have any routing information to the destination. Hence, the source node obtains the necessary path when required. A path may recruit multiple relays. During the signaling phase, the sender and the receiver can contract the relaying cooperative devices to function as relays. The selection of relays is based on the source flooding and the route discovery. In order to reduce the overhead generated during a route discovery phase, the "route cache" maintained at each node records routes the node has learned and overheard over a time frame. Each route discovered is stored in the route cache with a unique route index. So it is easy for us to pick multiple paths from the cache. To achieve high path independence, disjoint paths are preferred. PEMA needs to obtain multiple source–destination routes for a given source–destination pair, which is dependent on the node's ID. The multipath scheduling algorithm can be deployed at the source node to optimally split the data flow sent to a specific destination node. For delivering the real-time video streaming, we can use MPRTP (Multipath RTP) [31] in a video streaming service, in which the MPRTP data received from different paths are merged into a single bitstream and then decoded using a decoder. To provide reliable delivery, we can use MPTCP [2]. For simplicity, we only establish several standard TCP connections in parallel via multiple overlay paths in the subsequent simulation section.

For a more general case, other users can also set up similar multipath applications to form other overlay networks. These networks overlap each other and share the same underlay infrastructure resources, so their traffic may traverse the bottleneck links. If each overlay increases the path connection number endlessly, it is bound to cause the malignant resources occupancy. Therefore it is necessary to study how to build the multipath connections, and how to determine the number of paths to be established.

4. Model and problem statement

In this section, we formally introduce the multipath selection game, and then discuss how to quantify the network efficiency loss due to the behavior of selfish overlays.

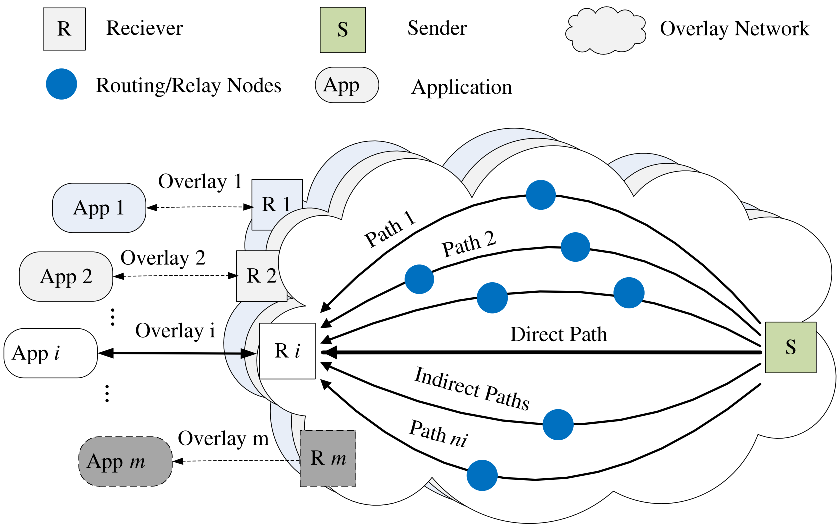

Fig. 3. Massively-multipath network model.

4.1. Model description

Suppose there are k ($k \geq 2$) end-to-end paths with different RTTs R_i ($\forall i \in \{1, 2, \ldots, k\}$) competing for the capacity or bandwidth C of a bottleneck link, as shown in Fig. 3. The situation of more than one bottleneck can be decomposed as several cases of single bottleneck, which can be studied individually. The overlay routing indicates the presence of redundant routes between a pair of hosts, which provides the ability to deliver the traffic along multiple paths simultaneously. Each single path congestion window grows and shrinks according to the special control policy. Note that the overlay is not a stable network shared by many end users, but rather a temporary self-organizing network built for once according to a user's application request. Each overlay adopts the principle of shortest path for each application request, thus it is "selfish".

We assume that there are m overlays, whose source nodes act selfishly and choose their paths with different numbers. We treat an individual overlay i as a player, whose strategy n_i is a feasible number of paths to be concurrently used. In practice, n_i takes positive integer value. However, in order to make our analysis tractable, we first treat n_i as a positive real-valued number, and then investigate the case where n_i is a positive integer number. Let S_i denote the feasible strategy set of player i, we have $n_i \in S_i$. In general, we assume that $S_i = [1, n_i^{\max}]$ (a closed and bounded real interval). In the rest of the paper, we will explicitly state a player's strategy set if it differs from $[1, n_i^{\max}]$. The game's feasible strategy space is represented as $S = S_1 \times S_2 \cdots \times S_m$. A feasible strategy profile is an m-dimension vector $n = (n_1, n_2, \ldots, n_m) \in S$.

The purpose of using multipath transfer by most overlay applications is to obtain high aggregate throughput. Let T_i denote the per-path throughput obtained by player i. Player i receives a payoff Ω_i that is a function of the aggregate throughput $n_i T_i$ (share of bottleneck link capacity) from all of its paths. We assume that the payoff function $\Omega_i(x)$ is an increasing and concave function of x. We also assume that a cost Φ_i is associated with all paths of player i. Note that when m is large enough, the throughput will not be increased significantly through adding a new path.

Note that all players must decide their choices simultaneously without communicating with each other. A best response, taking into consideration the strategies of other players, is a strategy that gives the most favorable outcome for a player. A Nash Equilibrium [7] is a solution where each player plays a best response to the strategies of other players. As an assumption, each player knows the strategies of other players, and no player can benefit from unilaterally changing its current strategy.

4.2. Utility functions

The utility U_i received by player i is a combination of its payoff and cost:

$$U_i(n_i) = \Omega_i(n_i T_i(n_i)) - \beta_i \Phi_i(n_i). \tag{1}$$

Note that an important application of the game is $\Omega_i(x) = x$, where a player's payoff equals to its received throughput [22,23]. Here, we use wastefulness coefficient $\beta_i \in [0, 1]$ to represent player i's degree on cost, that is, the weight of resource consumption cost for a certain benefit. A smaller β_i means fewer resources constrained or more computation resources saved for player i. These potential waste resources could be utilized by normal data transmission, so we use this metric to represent the efficiency in terms of cost for different overlay networks. In this paper, we study the case when m players compete for the capacity C, and the objective of player i is to maximize its utility U_i by adjusting n_i.

The game's main components are: the set of players, the set of actions and the set of payoffs. The players seek to maximize their utilities by choosing strategies that deploy actions depending on the available information at a certain moment. Each player chooses strategies which can maximize its utility. The combination of best strategies for each player is known as equilibrium. When each player cannot benefit anymore by changing its strategy, we say that the game's solution represents Nash Equilibrium. However, some games may not have a Nash Equilibrium or have more than one Nash Equilibriums.

Given that all players behave in a selfish manner, their interactions may or may not lead to a stable network state where all players are happy with their numbers of paths and no player wants to unilaterally deviate from it. Such a stable state is a Nash equilibrium $\mathbf{n}_{ne} = (n_1^*, \ldots, n_m^*)$, formally defined as: $n_i^* = \arg\max_{n_i \in S_i} U_i(n_1^*, n_2^*, \ldots, n_i^*, \ldots, n_m^*), \ \forall i \in \{1, 2, \ldots, m\}$. Similarly, we use p_{ne} to denote the network loss probability at an NE.

4.3. Throughput

In the following, we discuss the throughput and cost function. Player i's per-path throughput T_i is the packet successful delivery rate of one path, which is a product of per-path sending (or offered) rate B_i and network loss probability p. Namely, we have:

$$T_i = B_i(1 - p). \tag{2}$$

A well-known TCP sending rate model in [32] relates TCP sending rate B_i to loss probability p and RTT R_i, which is too complicated for tractable analysis. Therefore, we consider the following simplified version (recommended in the TFRC standard proposal [33]):

$$B_i = 1/\left(\mu R_i \sqrt{p} + T_{0,i}\nu\left(p^{3/2} + 32p^{7/2}\right)\right) \tag{3}$$

where $\mu = \sqrt{2b/3}, \ \nu = 3/2\sqrt{3b/2}, \ b = 1$ or 2, and $T_{0,i} = 4R_i$.

We relate throughput T to only network parameters. As all paths are expected to experience the same loss probability p at a congested bottleneck link, a common expected window size W^* can be assumed for all paths [29]. Then, the per-path throughput of player i is given by $T_i = W^*/R_i$. The bottleneck principle indicates that the sum of throughputs of all paths from all players equals to the bottleneck link capacity C, i.e.,

$$\sum_{i=1}^{m} n_i T_i = C. \tag{4}$$

Then, we have $W^* = C/\sum_{i=1}^{m} n_i/R_i$ and

$$T_i = (C/R_i) \bigg/ \sum_{i=1}^{m} n_i/R_i. \tag{5}$$

4.4. Network cost

Regarding the cost, we use the cost function to capture different player behaviors. In reality, an overlay needs to consume end host and network resources to generate data packets, so the cost is proportional to the packet transferring rate. Recall that for player i, the end host and network resources consumed by a single path to transfer data packets is proportional to B_i. Therefore the cost with respect to player i can be modeled by a utility function including payoff and cost n_iB_i. In this section, we focus on the case where a player's payoff equals to its total throughput. We rewrite utility function (1) as:

$$U_i(n_i) = \left[C\frac{n_i}{R_i} \bigg/ \left(\frac{n_i}{R_i} + \sum_{k=1, k\neq i}^{m} \frac{n_k}{R_k}\right)\right] - \beta_i n_i B_i. \tag{6}$$

The aggregate sending rate of all paths ($\sum_{i=1}^{m} n_iB_i$) drives the bottleneck link to full utilization. But not all data are eventually sent through the bottleneck link. Data packets are dropped with probability p (bottleneck link loss probability). Based on (2), we can rewrite (6) as:

$$\sum_{i=1}^{m} n_i B_i = C/(1 - p). \tag{7}$$

Eq. (7) indicates that a larger bottleneck link loss probability p corresponds to a larger total sending rate, which can also be interpreted as more network resources utilized. Thus, either p or equivalently $\sum_{i=1}^{m} n_iB_i$ can be thought of as a cost to the network. In other words, B_i can be thought of as a "cost" from a system's respective. Meanwhile, B_i can also be treated as a cost from a player's respective, because transferring data packets consumes end host and network resources.

In order to achieve the highest efficiency, a network operator may want to minimize the network cost [12,13] defined in (6) while maintaining the bottleneck link fully utilized. Therefore a system optimization problem is formulated as:

$$\min_n \Phi = \min_n \sum_{i=1}^{m} n_i B_i \tag{8}$$

$$\text{s.t.}: B_i = 1/\left(\mu R_i \sqrt{p} + T_{0,i}\nu\left(p^{3/2} + 32p^{7/2}\right)\right) \tag{9}$$

$$B_i(1-p) = (C/R_i) \Bigg/ \left(\sum_{j=1}^{m} n_j/R_j\right), \quad \forall i \tag{10}$$

$$n_i \in [1, n_i^{\max}]; \qquad m \geq 2; \qquad \mathbf{n} = \{n_1, \ldots, n_m\}. \tag{11}$$

It can be shown that p is an increasing function of $\mathbf{n}$, implicitly defined by (9) and (10). As the minimum p corresponds to the minimum cost, the optimal operating point of the whole system is the one where each player opens only one path (as shown in the following theorem). Note that in order to achieve the minimum cost, all players must be cooperative.

5. Nash equilibria of asymmetric game

In this section, we study the multipath selection game as an asymmetric selection game. After demonstrating the existence and uniqueness of an NE, we present a procedure for computing the NE.

5.1. Asymmetric game where overlays differ in RTTs

We first study the asymmetric game where overlays have different *RTTs* but same level of wastefulness. We rewrite utility function (6) as:

$$U_i(n_i) = \left[C\frac{n_i}{R_i} \Bigg/ \left(\frac{n_i}{R_i} + \sum_{k=1,k\neq i}^{m} \frac{n_k}{R_k}\right)\right] - \beta n_i B_i \tag{12}$$

where B_i is given in (9) and (10). For this asymmetric game we have the following result. We observe that utility function (12) is generally not concave. For example, suppose we consider a bottleneck link with bandwidth 10 Mbps and propagation delay 40 ms. Two overlays compete for this bottleneck link. Suppose overlay 2 has 10 paths and its average *RTT* is 100 ms. We vary the number of paths opened by overlay 1 whose average *RTT* is 150 ms. In Fig. 4, we see that the utility received by overlay 1 initially increases concavely as the number of paths increases, and decreases after reaching its maximum.

In the following, we show the existence and uniqueness of an NE in the game and describe a procedure for computing the NE.

(1) ***Nash equilibrium***

Theorem 1. *A unique Nash equilibrium (NE) exists in the asymmetric game in which all players differ only in their* RTTs. *This NE is an interior point in the strategy space of the game, provided that the number of players* $m < m_0$. *Threshold* m_0 *is the largest* m *satisfying*

$$m(1-p^*)B_1^* \leq C \tag{13}$$

where p^* *and* B_1^* *are respectively loss rate and the maximum overlay sending rate at the NE. At this interior-point NE, for any two players i and j, we have* $n_i^*/n_j^* = R_i/R_j$. □

The proof of the theorem is given in Appendix A. If we sort the numbers of paths in an ascending order as $n_1^*, n_2^*, \ldots, n_m^*$, then as we increase the number of players, all n_i^* will simultaneously decrease but maintain their relative proportional relationships. As m reaches a large enough number where n_1^* must be less than 1, player 1 will just maintain one path. From then on, as m continues to increase, NE will no longer be an interior point. To maintain an interior-point NE, m must be smaller than m_0, where m_0 is the largest m satisfying (13).

We can numerically solve for p^* and n_i^* at the NE as follows. Note that the unique NE is an asymmetric NE (each player has different number of paths at the NE). We take the first derivative of a player's utility function and set it to zero, we can get p^* and substitute it back into (10) to obtain n_i^*.

It is easy to see that at this interior-point NE, players have the same utility. And the efficiency loss of the NE is bounded. It is interesting to note that at the interior-point NE, players have the same utility and receive the same total throughput (the bandwidth share of the bottleneck link). This implies that the game does not favor those players with shorter *RTT*s (a network advantage). A player does not receive a higher throughput if it can afford to open a larger number of paths and/or has a smaller *RTT*. However, this is no longer true in another asymmetric game described in the following section.

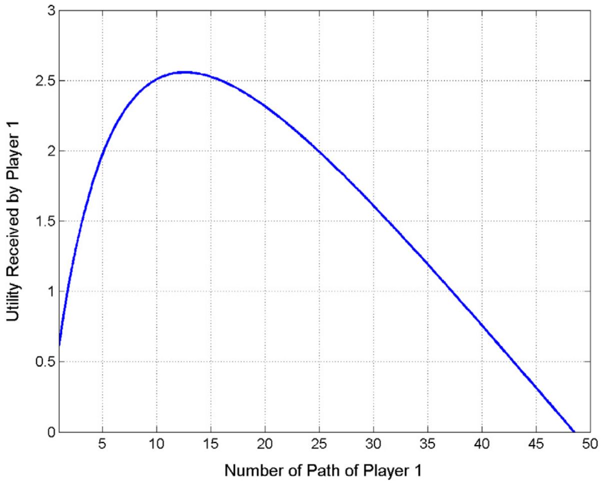

Fig. 4. Utility received by overlay 1 as a function of its number of paths.

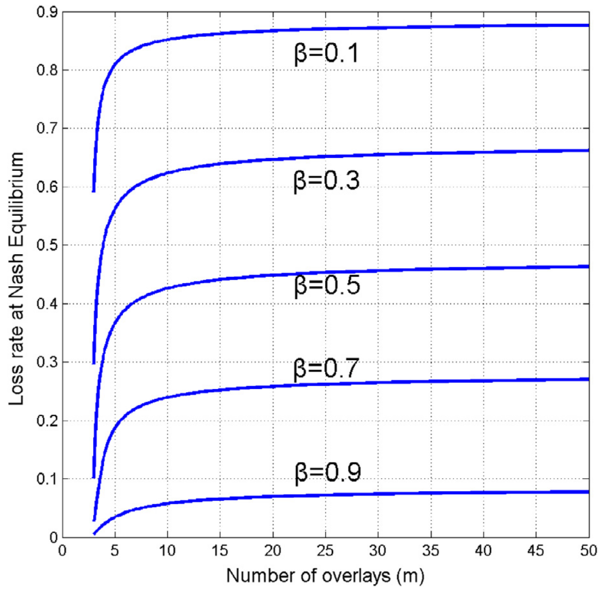

Fig. 5. p^* as a function of m for different β. m varies from 2 to 50.

In this example, we use the same network settings as before. We set $\beta = 0.3$ and the ith player with its $RTT = i \times 50$ ms. We vary the number of players m from 3 to 50, and all the players compete for a bottleneck link (with capacity $C = 10$ Mbps or 1250 pkts/s). In Fig. 5, for several different β values, we plot the loss rate p^* at NE when the number of players m increases. For any given β, we can see that p^* approaches to $1 - \beta$.

(2) ***Loss of Efficiency at the NE***

We use the loss of efficiency of NEs to quantify the network (or system) performance degradation due to the behavior of selfish players. The strict efficiency loss of an NE is defined as the ratio of the network cost at the NE and the minimum network cost. The worst efficiency loss is also called as "*price of anarchy*" [34]. Since there is a unique NE, the price of anarchy is just the efficiency loss of this unique NE. In this section, we first show that a network can perform arbitrarily badly at the NE (stable state). Then, we discuss the network efficiency loss at the NE compared to the optimal network state where the total system cost is the minimum.

As discussed in Section 4.3, p is an increasing function of the number of paths n. Therefore, an NE vector $\mathbf{n}_{ne} = (n_1^*, \ldots, n_m^*)$ corresponds to a unique p_{ne} which asymptotically approaches to 1 as n_i goes to 1. Therefore $p^* \to 1$ as $n_i^{\max} \to 1$. The network cost at the NE is: $\Phi_{ne} = \sum_{i=1}^{m} n_i^* B_{i,ne} = C/(1 - P_{ne})$. Let p_{opt} denote the loss probability with respect to the minimum network cost Φ_{opt}, which is a constant for a fixed number of players.

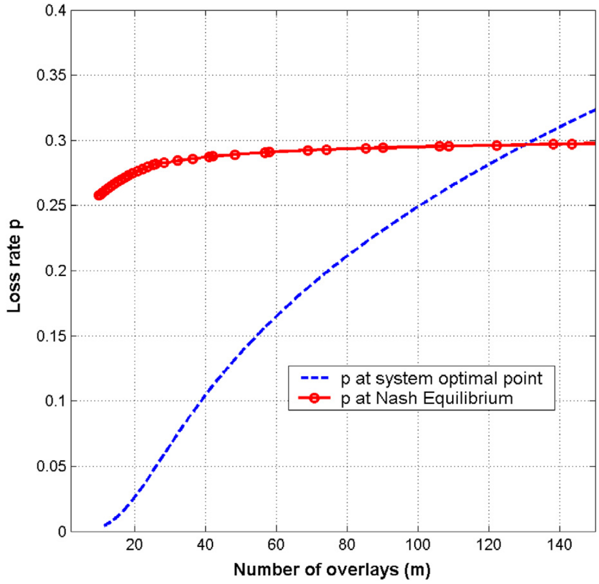

Fig. 6. Loss rate as a function of number of players m when $\beta = 0.7$.

Theorem 2. *In an asymmetric multipath selection game, the efficiency loss of the unique NE is upper bounded by* $(1-p_{opt,m=2})/\beta$*, where* $p_{opt,m=2}$ *denotes the loss rate when there are two players and each player opens only one path. This upper bound is a function of* C, R_1, R_2*, and* β*.*

Proof. Consider that the system cost when the system is at NE: $\Phi_{ne} = \sum_{i=1}^{m} n_i^* B_{i,ne} = C/(1-P_{ne})$, and let $\Phi_{opt} = C/(1-p_{opt})$ denote the minimum network cost. Then the loss of efficiency of an NE is given as: $L_{\text{eff}} = \Phi_{ne}/\Phi_{opt} = (1-p_{opt})/(1-p_{ne})$. Note that the loss of efficiency is always larger than or equal to 1. Recall that p_{opt} must satisfy $(1-p)m/\phi = C$, and p^* must satisfy $(1-p^*)m\,\mathbf{n}_{ne}/\phi^* = C$. Then we have $(1-p)/\phi = \mathbf{n}_{ne}(1-p_{ne})/\phi^*$. Since $\mathbf{n}_{ne} \geq 1$, we have: $p^* \geq p_{opt}$. As m increases, $\mathbf{n}_{ne}$ decreases. Before $\mathbf{n}_{ne}$ reaches 1, p^* is strictly larger than p_{opt}. Afterwards, $p^* = p_{opt}$. Then, the efficiency loss attains its maximum when m is small, and the efficiency loss is lower and is bounded by one. Since the proof of Theorem 1 indicates that $1 - p^* > \beta$ and p_{opt} is an increasing function of m, we have:

$$L_{\text{eff}} = (1-p_{opt})/(1-p^*) < (1-p_{opt,m=2})/\beta \tag{14}$$

where $p_{opt,m=2}$ denotes the loss rate when there are only two players and each player opens only one path (i.e., the network cost is the minimum based on Theorem 1 in [29]). Note that the upper bound in (14) is just a simple function of network parameter C, R_1, R_2 and the player's wastefulness coefficient β. □

By using the similar network settings as before, we look at the loss of efficiency as m increases. In this model, the player uses the greedy path selection to deploy its traffic onto the overlay paths, whose performance has been proved to be worse than that of the optimal selection. Yet, a Nash equilibrium point will be achieved through the sub-optimal strategy, and the Nash equilibrium (stable state) point may not be Pareto optimal.

In Fig. 6 we plot the loss rate at the NE and system optimal points. Recall that loss rate p is an increasing function of the number of paths $\mathbf{n}$, implicitly defined by (9) and (10). Therefore, an NE vector $\mathbf{n}_{ne} = (n_1^*, \ldots, n_m^*)$ corresponds to a unique p_{ne}. Let p_{opt} denote the loss rate corresponding to the optimal point. As all paths are expected to experience the same loss rate p at a congested bottleneck link, we can assume a common throughput for all paths. A larger loss rate p corresponds to a larger number of paths, which can also be interpreted as a higher total transfer rate. As the selfish player wishes to get high transfer rate in nature, it uses excessive number of paths and occupies more network resources, which may deteriorate the packet loss rate. As predicted, the loss rate at the NE is always greater than and equal to the loss rate at the system optimal point. When m is sufficiently large, the optimal point of the entire system is the one where all players use one path at the NE. Note that in order to achieve the minimum cost, all players must be cooperative. And the trajectory of loss rate increase is the same as that of system optimal point.

(3) ***Effect of overlay wastefulness***

In this section, we consider the case where all overlays have different *RTTs* but the same level of wastefulness (i.e., the same β). We vary β to see how the wasteful level affects the network performance at the NE. The case where overlays have different wastefulness is discussed in Section 4.2.

According to the definition of β, a larger β is used to characterize a more wasteful or a more resource-constrained player. As β gets larger, the players are likely to be more wasteful, which means the whole efficiency is deteriorated. Meanwhile, the

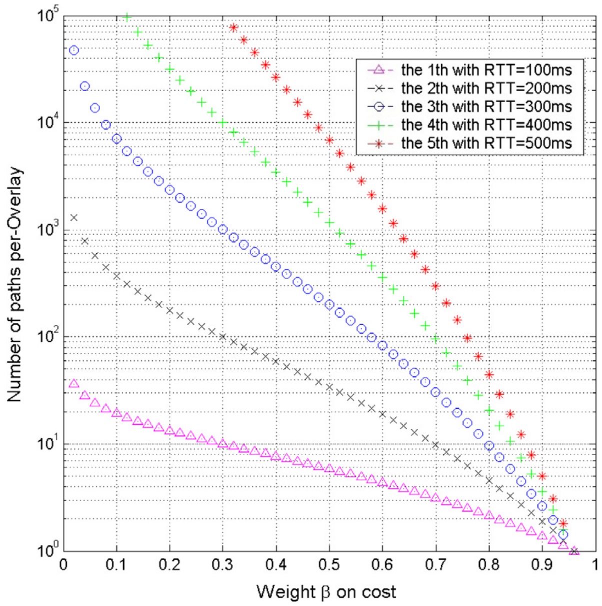

Fig. 7. Number of paths at NE as a function of β when $m = 5$.

overall network resource is a fixed value, and every player is only trying to reduce the cost for maximizing (12) in equilibrium. As the increased number of paths will increase the cost, the number of paths at the NE will be smaller. Intuitively, the packet loss is another waste, so the loss rate at the NE needs to be reduced to balance the efficiency of data packets. This intuition is indeed true and can be verified by looking at the relationship between the loss rate p^* at the NE and β.

Theoretically, each player i tries to obtain its optimal strategy n_i^*, as a response to the strategies of all other players. Thus, there is an interior stationary point NE $\mathbf{n}_{ne} = (n_1^*, \ldots, n_m^*)$, and $\forall i,\ \partial U_i/\partial n_i^* = 0$. Since the proof of Theorem 1 indicates Eq. (A.9), we can derive: $\partial p/\partial \beta < 0$. Hence p is a decreasing function of β. As $\mathbf{n}_{ne}$ is an increasing function of p, the players connect fewer and fewer paths at the NE as β increases. Recall that a smaller p corresponds to a smaller system or network cost, because the system cost at the NE is $\sum_{i=1}^{m} n_i^* B^* = C/(1-p_{ne})$. Therefore, we also expect that the system cost decreases as β increases. In addition, since p_{opt} is independent of β, we see that $L_{\text{eff}} = (1-p_{opt})/(1-p_{ne})$ is a decreasing function of β. That is, as players become more resource-constrained (larger β), the NE will be less efficient from a system's perspective.

As an example, we evaluate the performance of $m = 5$ players with the same β, and the ith player with its $RTT = i \times 100$ ms. All the players compete for a bottleneck link (with capacity $C = 10$ Mbps or 1250 pkts/s). We expect the loss rate at the NE to decrease as β increases, and finally reach the system optimal point. This is indeed true, as shown in Fig. 7. β is so large that the NE is no longer an interior point of the strategy space, and every player is so conservative and opens just one path. Fig. 8 shows that the loss of efficiency decreases as players become less and less wasteful, as expected.

5.2. Asymmetric game where overlays differ in wastefulness

We study another asymmetric game where overlays have the same RTT but different levels of wastefulness. We show the existence of an NE. Contrary to the previous asymmetric game, we show that a more wasteful player has a larger utility and a larger bandwidth share of the bottleneck link than a more saving player.

We rewrite a player's utility as follows:

$$U_i(n_i) = \left[Cn_i \bigg/ \left(n_i + \sum_{k=1,k\neq i}^{m} n_k \right) \right] - \beta_i n_i B. \tag{15}$$

Note that in this utility function, the per-path offered rate B is the same for all players as they all have the same RTT. For this game, we can show the existence of an NE. In order to find an NE, we need to solve the following equations:

$$C\phi - (1-p)\sum_{j=1}^{m} n_j = 0 \tag{16}$$

$$\beta_i n_i - n_{-i}(1-p-\beta_i)[\varphi(1-p)/\phi + 1] = 0 \tag{17}$$

$$\forall i \in \{1, 2, \ldots, m\}, \quad \text{where } n_{-i} = \sum_{j=1,j\neq i}^{m} n_j$$

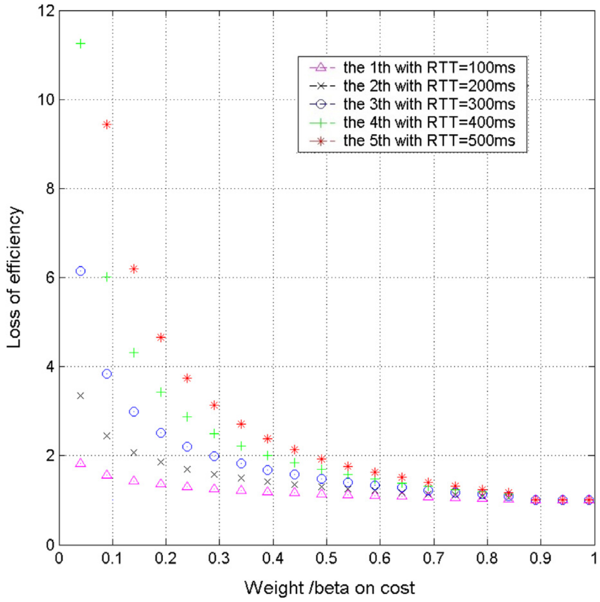

Fig. 8. Loss of efficiency at NE as a function of β when $m = 5$.

where (16) states that the total throughput equals to the bottleneck link capacity, and (17) can be obtained by setting the derivatives of utility (with respect to the player's number of paths) to zero.

From these equations, we have the following necessary condition for the existence of NE $(\mathbf{n}_{ne}, p^*)$:

$$n_i = \frac{C\phi/(1-p)(1-p-\beta_i)[\varphi(1-p)/\phi+1]}{\beta_i+(1-p-\beta_i)[\varphi(1-p)/\phi+1]}. \tag{18}$$

Substituting (18) back into (16) yields an equation for the loss rate p^* at NE. Then a sufficient condition for having a valid solution p^* can be derived. Hence, the existence of NE can be proved. Once we obtain p^*, we can obtain $\mathbf{n}_{ne}$ through (18).

Then, a player's utility at NE is given by:

$$U_i(n_i) = Cn_i \Big/ \sum_{j=1}^{m} n_j - \beta_i n_i B = \frac{Cn_i/(1-p)[\varphi(1-p)/\phi+1]}{\frac{\beta_i}{(1-p-\beta_i)^2}+\frac{\varphi(1-p)/\phi+1}{1-p-\beta_i}}. \tag{19}$$

From (19), we can see that a more wasteful (β) player has a larger utility and a larger throughput (bandwidth) share than a less wasteful player. In other words, more wasteful selfish TCP overlays are better off than more saving overlays in a stable network state. The result is different from the one obtained in the case where overlays differ only in their *RTT*s in the game.

5.3. Stability of NE in best-response dynamics

In this section, we further study some dynamic aspects of Nash equilibrium. Specifically, we are interested in a dynamic game playing process, called best-response dynamics [5]. According to this dynamics, the game starts from an initial state, and then follows a prescribed moving sequence where players take turns making optimal moves or best responses to the other players. An interesting question is whether this process eventually converges to an NE.

An NE is stable if it can be achieved in such a best-response dynamics. Specifically, as defined in [5], after some player deviate from an NE in the feasible strategy set, all players adjust their responses optimally based on some fixed ordering of moves, and if this process converges to the original NE, then we say that this NE is globally stable with respect to this adjustment scheme. In addition, we define the local stability of an NE by restricting the domain of the deviation to the ε-neighborhood of the NE. Section 4.3 in [5] shows that the stability conditions of NEs are the same regardless of the adjustment schemes when there are only two players.

Here, we only study the stability of the NE in a two-player variant of Asymmetric Game under the best-response dynamics. We can solve each player's optimization problem to obtain its response or reaction function as $n_1(t+1) = f_1(n_2(t))$ and $n_2(t+1) = f_2(n_1(t))$ where t indicates the discrete time step.

In the numerical simulations, we observe that the best response functions of both players are always concave for a large range of network parameters. We can show that the concavity of the best response functions and the uniqueness of an NE imply the global stability of the NE. Since this concavity property together with the uniqueness of NE indicates the global stability of NE, we conjecture that it is very likely that the multipath selection game has a global stable NE and the best-reply dynamics always converges to the NE.

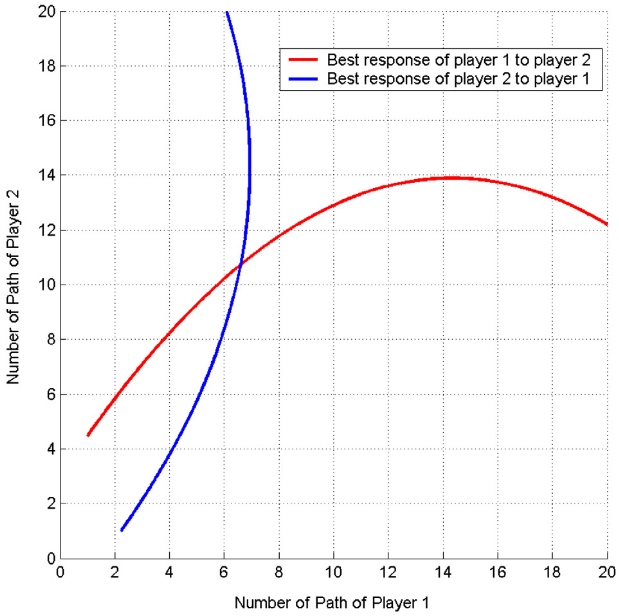

Fig. 9. Existence of Nash equilibrium in a two player game.

We now use an example to illustrate the existence of an NE and the stability of the asymmetric game in Section 5.2 where overlays have different levels of wastefulness. Suppose two players compete for a bottleneck link with 10 Mbps capacity and both of them have the same *RTT* of 200 ms. Player 2 with $\beta_2 = 0.6$ is less wasteful than player 1 with $\beta_1 = 0.9$. A Nash equilibrium can be identified in the best response curves in Fig. 9. Note that we can graphically identify an NE of a two-player game by plotting the best response curves of both players. For example, consider that player 2 tries to find the best number of paths $f_2(n_1)$ that maximizes its utility for any given n_1. We can plot player 2's best response curve by computing $f_2(n_1)$ for a range of n_1. Similarly, we can plot player 1's best response curve $f_1(n_2)$. The intersection point of these two curves is an NE. For the simplified TCP model, we can use some optimization tools (e.g., the optimization toolbox in Matlab) to compute the best responses. In Fig. 9, we can see that there is an NE in this game where players have different levels of wastefulness. Player 2 is less wasteful or less resource constrained than player 1, namely, $\beta 1 > \beta 2$. The result shows that the more wasteful player 1 has less than 7 paths, while the less wasteful player 2 has more than 10 paths at the NE. This means the throughput achieved by player 2 is more than that of player 1.

5.4. Integer asymmetric game

We observe that the number of paths of a player at an NE in previous asymmetric games takes continuous non-integer values. However, in practice, the numbers of paths opened by a player can only take integer values. This motivates us to consider another interesting variant of the multipath selection game where each overlay can only choose a positive integer number of paths. That is, each player's strategy set is $\mathbf{N}$. We assume that all players have different utility functions. We call this the *Integer Asymmetric Game*.

In order to study this game, we use the results of the corresponding continuous asymmetric game derived in Section 5. Note that if the pure-strategy NE in the continuous game is an integer vector, then it must be an NE in the corresponding integer game. However, if the pure-strategy NE of the continuous game is a non-integer vector $\mathbf{n}_{ne} = (n_1^*, \ldots, n_m^*)$, we can approximate n_i^* by taking floor n_{if}^* and ceiling n_{ic}^* of n_i^* to obtain the 2^m integer-valued vectors. Next, we demonstrate that this integer game must have pure-strategy NE(s) at some of these integer vectors. In the following, we call such vectors *approximate Nash Equilibrium* (differ from the ε-NE defined in [35]).

We can see that, at these approximate NEs, the utility deviation of each player from that in the non-integer real NE is bounded. In other words, all approximate NEs asymptotically approach the unique NE of the continuous asymmetric game as the number of overlays increases. Then the important question is whether these approximate NEs are real NEs in the integer game? We demonstrate in the following theorem that some of these approximate NEs are pure-strategy NEs in the integer game under some conditions.

Theorem 3. *In an integer asymmetric multipath selection game, if the Nash Equilibrium of the corresponding continuous asymmetric game is a non-integer vector $\mathbf{n}_{ne} = (n_1^*, \ldots, n_m^*)$ with n_{if}^* and n_{ic}^* respectively denoting the floor and ceiling of n_i^*, then the integer game admits a pure-strategy integer Nash Equilibrium provided that the following condition is satisfied: the*

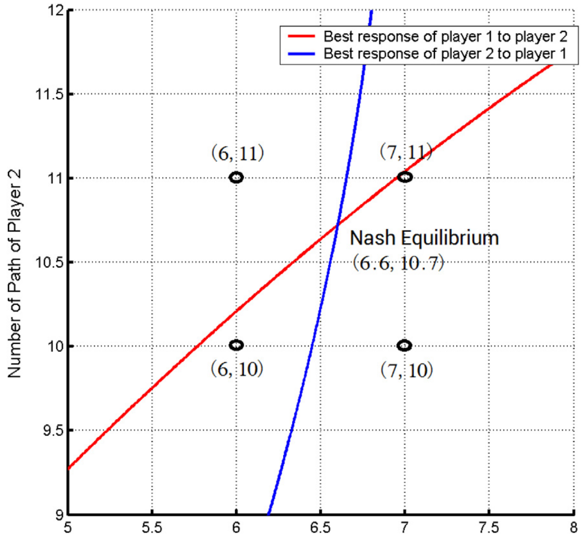

Fig. 10. An observed case where integer NE must exist.

best response for player i ($\forall i \in \{1, 2, \ldots, m\}$) is always chosen from n^*_{if} and n^*_{ic} given that all other players choose either n^*_f or n^*_c.

Proof. We can form a new game G by restricting each player to have only two strategies n^*_{if} and n^*_{ic}. That is, if the original integer game has a positive integer strategy set $\mathbf{N}$ for each player, then in game G we have $S_G = \{n^*_{if}, n^*_{ic}\}$ for each player. We claim that any NE of G must be an NE of the original integer game. If we pick any NE $\mathbf{n}^*_G = (n^*_{1,G}, \ldots, n^*_{m,G})$ of game G, we know that $U_i(n^*_{i,G}, n^*_{-i,G}) \geq U_i(n_{i,G}, n^*_{-i,G})$; $n_{i,G} \in S_G$ where $\mathbf{n}^*_{-i,G} = (n^*_1, \ldots, n^*_{i-1}, n^*_{i+1}, \ldots, n^*_m)$ and $n^*_{k,G} \in S_G$, $\forall k$. Based on the assumption, we immediately know that $U_i(n^*_{i,G}, n^*_{-i,G}) \geq U_i(n_{i,G}, n^*_{-i,G})$ if $n_i \in \mathbf{N}$, $\forall i$. This implies that $\mathbf{n}^*_G$ must be an NE in the original integer game. In fact, there must be an NE in any asymmetric game with each player having only two strategies [36]. Thus, we conclude that, in the original *integer* game, there must be a pure-strategy NE to be achieved at some vectors formed by n^*_{if} or n^*_{ic}. □

Now consider an example of a two-player integer game. The simulation environment configuration is the same as Section 5.3. The best response curves in the continuous version of the game are plotted in Fig. 9. We see that there is indeed one unique NE (6.6, 10.7) in Fig. 10. If we restrict the strategy space to the set of positive integers, we can approximate the continuous game NE with the floors and ceilings of the NE vector to obtain four approximate NEs: (6, 10), (6, 11), (7, 10) and (7, 11).

To illustrate Theorem 3, we observe that in Fig. 10 when player 2 chooses 11 as its strategy, the best integer response of player 1 must be either 6 or 7. This is because for each strategy of player 2, player 1 has a unique best response. All other strategies monotonically decrease the utilities as they deviate from the best response, since there is only one unique interior-point maximum for the utility function (15). Similarly we have the following arguments: when player 2 chooses 10, the best *integer* response of player 1 must be either 6 or 7; and when player 1 chooses 6 or 7, the best *integer* response of player 2 must be chosen from 10 and 11. This observation is exactly the condition required for Theorem 3.

It can be seen from the best response curves in Fig. 9, there is a fractional NE in the continuous game. If we assume that the integer strategy closest to the response curve uses the integer with the highest utility among all integer strategies, then we can see that the game has the no pure-strategy integer NE. However, we never saw this case in our simulations. Actually, since the condition required for Theorem 3 is satisfied in all of our simulations, we conjecture that pure-strategy integer NE always exists in the integer game.

It is interesting to note that in the integer game, a *mixed-strategy NE* always exists if players' strategy spaces are bounded. Recall that a mixed strategy of a player is defined as a probability distribution over the player's pure-strategy space. Suppose that in the *integer asymmetric game*, each player has a maximum allowable number of paths. As each player can only choose an integer number of paths, the integer game is a finite game in which each player has only finite choices in its strategy space. As proved by Nash [37], if players are allowed to use mixed strategies, then every n-player finite game admits at least one mixed-strategy Nash Equilibrium. Hence, if we put an upper bound on each player's strategy set, there must exist at least one mixed-strategy Nash Equilibrium in the integer game.

6. A general case considering path cost

Recall that in Section 4 the cost of players only considers the packet transferring cost. In this section, we introduce a cost that is specific to the paths, which accounts for the resource consumption of maintaining connected paths. Specifically, we use $\alpha_i n_i$ to represent the cost that is proportional to the number of paths connected by player i, and refer to α_i as the computation power coefficient of player i. The cost $\alpha_i n_i$ can be thought of as the host and network resources used to maintain n_i paths. Hence, we refer to this cost as the path cost.

We now consider a generalized cost function where a player's total resource consumption includes both the packet transferring cost and the path cost. We study the case where a player's payoff equals to its total throughput and all players have the same utility function. We can rewrite utility function (1) as:

$$U_i(n_i) = \left[Cn_i \Big/ \left(n_i + \sum_{k=1,k\neq i}^{m} n_k \right) \right] - \beta_i n_i B - \alpha n_i \tag{20}$$

where $\alpha_i \neq \alpha_j, \forall\, i \neq j$. Recall that $V_i(x)$ is the payoff received by player i when it receives a throughput x. $V_i(x)$ is a continuous, concave and increasing function of x. Here we consider a symmetric "general game", and all players have the same per-path sending rate B as they have the same *RTT*. We have the following theorem for this game.

Theorem 4. *A symmetric "general game" admits a unique Nash equilibrium (NE) $\mathbf{n}_{ne}$ in pure strategies. At the NE, all players have the same number of paths. The NE is an interior point of the strategy space for $m < m_{0,\alpha}$ and $\mathbf{n}_{ne} = (1, 1, \ldots, 1)$ for $m \geqslant m_{0,\alpha}$. Note that $m_{0,\alpha}$ is the largest m such that $m(1 - p_a^*)/\phi_a^* \leqslant C$, p_a^* is the loss rate at the NE, and ϕ_a^* is a function of p_a^*, defined in (13) in Section 4.1.*

Following a similar procedure as used in the proof of Theorem 1, we can show the existence of a unique NE $\mathbf{n}_{ne} = (n_\alpha^*, n_\alpha^*, \ldots, n_\alpha^*)$ and obtain $m_{0,\alpha}$. See Appendix B for details.

Since at the NE, we must have $(1-p_a^*)/\varphi_\alpha^* = C/(m\,\mathbf{n}_{ne})$, and all players must have at least one path opened, i.e., $\mathbf{n}_{ne} \geqslant 1$, we must make sure $m \leqslant m_{0,\alpha}$ where $m_{0,\alpha}$ is the largest m such that

$$m(1 - p_{ne})/\varphi_\alpha^* \leqslant C. \tag{21}$$

We can use the numerical method to identify $m_{0,\alpha}$. Similar to Theorem 1, we have $p_a^* < p_{0,\alpha}$ where $p_{0,\alpha}$ is the solution of $1 - p_{0,\alpha} = \alpha\varphi_{0,\alpha} + \beta$. Thus, $1 - p_a^* > \beta$. We know that as m increases, $p^* \to p_{0,\alpha}$ and φ_α^* also increases to $\varphi_{0,\alpha}$ (function of $p_{0,\alpha}$). Thus, $(1 - p_\alpha^*)/\varphi_\alpha^*$ is bounded. So, when m becomes larger and larger, eventually, $m(1 - p_a^*)/\varphi_\alpha^*$ will be larger than C, which means that all players only use one path at the NE.

We observe that players in the "general game" are more conservative than players in "special game", as in "general game" a player's cost includes both packet transferring cost and path cost. Hence, we expect that at the NE of "general game", players open smaller number of paths than at the NE of either asymmetric or symmetric "special game".

Proposition 1. *The interior-point Nash equilibrium (NE) of symmetric "general game" yields a smaller loss rate and a smaller number of paths than the NE of "special game".*

Recall that the loss of efficiency of the multipath selection game is $L_{\text{eff}} = (1 - p_{opt})/(1 - p_{ne,\alpha})$. The loss of efficiency of the NE in symmetric "general game" is always larger than or equal to one, but it is upper-bounded. Compared with "special game", the actual efficiency loss of symmetric "general game" is smaller because these players are more conservative. For example, the loss of efficiency of NE in symmetric "general game" is always larger than or equal to one, but it is upper-bounded. The proof of Theorem 4 shows that $1 - p^* > \beta$. Since p_{opt} is an increasing function of m, we have $L_{\text{eff}} = (1 - p_{opt})/(1 - p^*) < (1 - p_{opt},\ m = 2)/\beta$.

Even though this upper bound of symmetric "general game" is the same as that of "special game", we can see that the actual efficiency loss of this game is smaller than that of "special game" since $p_a^* < p^*$. The result in this section again indicates that we might not expect a large efficiency loss or congestion collapse in reality.

7. Simulation and evaluation results

In this section, we verify our analytical results through OPNET simulations. We consider a single bottleneck link with capacity 10 Mbps or 1250 pkt/s and with 50 ms delay, competed by $m = 10$ players. These players are allowed to use multiple paths, and the RTT of the ith player is $i \times 50$ ms. We vary the buffer size and the number of paths for each scenario. The bottleneck link router uses FIFO scheduling, and its buffer management is *Drop-Tail*. The TCP segment size is set to 500 bytes, and each sender has unlimited amount of data to send.

Note that all of our analytical results are based on the TFRC model. Therefore, it is important to know how well this model captures real TCP behavior when the number of paths is very large (which is the scenario in the multipath selection game). We show respectively in Figs. 11 and 12 the loss probabilities and throughputs that are measured in OPNET simulations, estimated by TFRC (TCP Friendly Rate Control) model [33], and estimated by TCP New Reno model [8]. Note that in order to get the loss probability estimated by TFRC model in Fig. 11, we use the measured TCP sending rate B_i to numerically solve for the estimated loss probability p based on the TFRC model (3). We also use B_i to solve for the estimated loss probability p based on the TCP New Reno model. Fig. 11 shows that the TFRC model always gives a better estimate of loss probability than the TCP New Reno model. Note that the TFRC model slightly over-estimates the loss probability when the number of paths

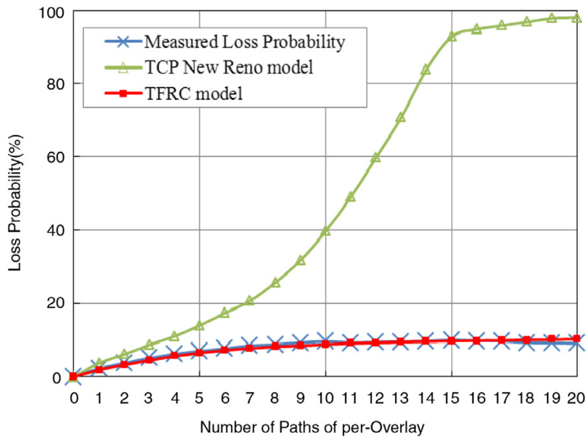

Fig. 11. Per-path loss probability.

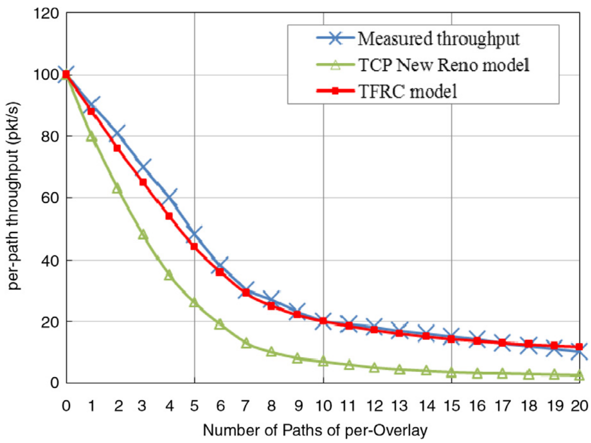

Fig. 12. Per-path throughput.

is more than 15. As an over-estimated loss probability implies more conservative behaviors of players, players are likely to have smaller numbers of paths at NEs than those obtained by our analytical results that are based on the TFRC model. But all other analytical results still hold.

In addition, due to the global synchronization, all the paths share the resource very fairly: in the steady state they experience the same losses and transmit the same number of packets. We can aggregate all the paths as one big path. Fig. 12 shows that the TFRC model gives a very good estimate of the measured per-path throughput.

In Fig. 13, we use OPNET simulations to illustrate the existence of a unique Nash equilibrium in a two-player continuous asymmetric game. Both players have the same two-way propagation delay 40 ms. The bottleneck link queue is a RED queue with a target queuing delay 10 ms. Both players use utility function (given in (6)): player 2 has the wasteful coefficient $\beta_2 = 0.6$ and player 1 has $\beta_1 = 0.9$. This means that Player 1 is more wasteful than player 2. Fig. 9shows that the predicted NE by our analysis is very close to the one obtained from OPNET simulations in Fig. 13.

8. Conclusions

In this paper, we studied the collaboration among selfish overlays in a pervasive computing infrastructure, where each overlay can open multiple concurrent paths to maximize its individual throughput or other utilities for the purpose of improving its quality of experience. In order to study how the selfish overlay's behavior impacts on the stability and efficiency, we modeled overlays as players in a non-cooperative game competing for the capacity of a bottleneck link, and we referred to this game as the multipath selection game with the objective of maximizing the throughput. We used the well-known TFRC model in our analysis. Our game-theoretic analysis demonstrated the existence or uniqueness of Nash equilibrium in this multipath selection game. Furthermore, we discussed the asymmetric case where all overlays have different *RTTs* and different wastefulness levels β. We found that overlays differing only in their *RTT*s still receive proportional throughput shares and utilities at the NE, and more wasteful selfish overlays are better off than more saving overlays in a stable network state. Interestingly we found that pure-strategy integer NEs always exist when asymmetric

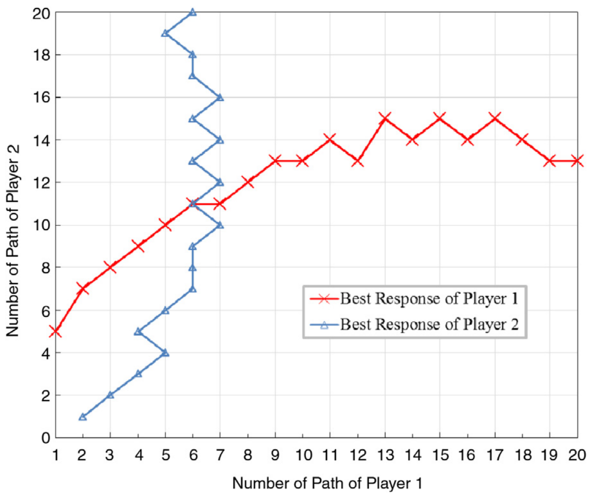

Fig. 13. Nash equilibrium observed in OPNET simulation.

overlays are restricted to open only integer numbers of paths. For the general case, we found that overlays open smaller numbers of paths than at the NE of either asymmetric or symmetric "special game", and the actual efficiency loss of this game is also smaller than that of "special game".

The proposed PEMA can be conveniently deployed over the existing IP networks, which can reduce the impact of information explosion on enterprise content servers in the cloud and subsequently open the door to new pervasive multimedia services. Many interesting issues could be explored in future. For example, it would be very interesting to study a game with more than one network bottlenecks in practice. It would also be interesting to further our study on the asymmetric game by considering that overlays have different service requirements. Furthermore, we plan to further explore the cooperative game model, because the SDN-like (Software-Defined Networking) centralized network architecture provides the convenience and possibility for the unified control and judgment.

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Appendix A

The appendix provides the proof of Theorem 1. Each player i tries to solve for its optimal strategy n_i^*, as a response to the strategies of all other players. Thus, if there is an interior point NE $\mathbf{n}_{ne} = (n_1^*, \dots, n_m^*)$, then it must be true that $\forall i$, $\partial U_i/\partial n_i^* = 0$, and $n_i^* = R_i \arg\max_{n_i \in S_i} U_i(n_1^*, n_2^*, \dots, n_i^*, \dots, n_m^*)$. If we denote $z_i = n_i/R_i$, utility function (6) is rewrited as:

$$U_i(z_i) = \left[Cz_i \Big/ \left(z_i + \sum_{k=1, k\neq i}^{m} z_k \right) \right] - \beta z_i B_i. \tag{A.1}$$

In the following, we first introduce a fact indicating that the stationary point satisfying $\partial U_i/\partial z_i = 0$ is actually the maximum point if it is in $[1, \infty)$. Then we show that there is a unique $\mathbf{n}_{ne}$ satisfying $\partial U_i/\partial z_i^* = 0, \forall i$.

First, we need to seek all vectors $\mathbf{n}_{ne}$ satisfying a set of m equations

$$\partial U_i/\partial z_i = 0, \quad \forall i \in [1, 2, 3, \dots, m]. \tag{A.2}$$

We first prove that if $\mathbf{n}_{ne}$ exists, $n_i^*/n_j^* = R_i/R_j, \ \forall i, j$. Then, we show that $\mathbf{n}_{ne}$ is actually unique by proving that there is only one p^* such that $n_i^*/n_j^* = R_i/R_j, \ \forall i, \ j$.

To simplify calculation formula, first we need to set $\bar{\phi} = \mu\sqrt{p} + 4\nu\left(p^{3/2} + 32p^{7/2}\right)$, thus $B_i = 1/R_i\bar{\phi}, \ \phi_i = R_i\bar{\phi}$ and $\varphi_i = R_i\overline{\varphi}$. As $B_i = C/R_i\,(1-p)\sum_{i=1}^{m} n_i/R_i$, we have $C\bar{\phi} = (1-p)\sum_{i=1}^{m} n_i/R_i$, and utility function Eq. (A.1) is further rewrited as:

$$U_i(n_i) = [Cz_i/(z_i + z_{-i})] - \beta_i z_i/\bar{\phi} = [Cz_i/(z_i + z_{-i})] - C\beta_i z_i/\,(1-p)\,(z_i + z_{-i}).$$

For an arbitrary player i, we have:

$$\frac{\partial U_i}{\partial z_i} = \frac{Cz_{-i}}{(z_i+z_{-i})^2} - \frac{\beta}{\bar{\phi}} + \frac{\beta n_i \frac{\partial \bar{\phi}}{\partial p}}{\bar{\phi}^2}\frac{\partial p}{\partial z_i} = \frac{Cz_{-i}}{(z_i+z_{-i})^2} - \frac{\beta}{\bar{\phi}} - \frac{\beta z_i C\varphi}{(z_i+z_{-i})^2[(p-1)\varphi - \bar{\phi}]} \tag{A.3}$$

where $\bar{\phi} = \phi_i/R_i = \mu\sqrt{p} + 4\nu\left(p^{3/2} + 32p^{7/2}\right) = 1/R_iB_i$

$$\overline{\varphi} = \varphi_i/R_i = \frac{\mu R}{2\sqrt{p}} + T_0\nu\left(\frac{3}{2}\sqrt{p} + 112p^{5/2}\right).$$

And $z_{-i} = \sum_{k=1,k\neq i}^{m} z_k$ and $\overline{\varphi} = d\bar{\phi}/dp$, $\mu = \sqrt{2b/3}$, $\nu = 3/2\sqrt{3b/2}$, $b = 1$ or 2, and $T_{0,i} = 4R_i$.

Fact 1. Best response of a player is unique, which is the stationary point if the stationary point is in $[1, \infty)$. First we need to show that for any given z_{-i}, there is only one unique maximal point for U_i. In fact, player i needs to solve the following equations to get a candidate for a maximal point z_i^m:

$$0 = \beta z_i - z_{-i}(1-p-\beta)[\overline{\varphi}(1-p)/\bar{\phi} + 1] \tag{A.4}$$

$$0 = C\bar{\phi} - (z_i + z_{-i})(1-p) \tag{A.5}$$

where Eq. (A.4) is a simplification of $\partial U_i/\partial z_i = 0$. We can regard z_i^m and p as the implicit functions of z_{-i}. Note that for any given z_{-i}, there exists a unique pair of (z_i^m, p) as the solution to Eq. (A.4) and Eq. (A.5). In fact, the unique stationary point z_i^m obtained from this implicit function is indeed a maximal point. We can enlarge the domain of U_i to be $(0, \infty)$, and notice that z_i^m is also a unique stationary point for this enlarged domain. Since $U_i(0, z_{-i}) = 0$ and $\lim_{z_i\to\infty} U_i = -\infty$, the values of two end points are not larger than $U_i(z_i^m, z_{-i})$ given that z_i^m is indeed an interior point. Then we can conclude z_i^m is indeed a maximal point in domain $(0, \infty)$. If z_i^m is still a stationary interior point in $[1, \infty)$, then it also must be a maximal point. We can show that $z_i^m = f_i(z_{-i})$ and $p = f_p(z_{-i})$ are continuous functions on domain $z_{-i} \in (0, \infty)$, in which $f_i(z_{-i})$ is referred to as the best response function. In addition, from implicit function theorem, we know that they are continuously differentiable.

Now, we go on to prove the existence and uniqueness of NE. Consider two arbitrary players i and j, and let $\delta_i z_i = \sum_{k=1,k\neq i}^{m} z_k$; $\delta_j z_j = \sum_{k=1,k\neq j}^{m} z_k$.

When $\partial U_i/\partial z_i = \partial U_j/\partial z_j = 0$, we have:

$$(1-p)[\delta_i + \beta\overline{\varphi}/((1-p)\overline{\varphi} + \bar{\phi})]/(1+\delta_i) - \beta = 0 \tag{A.6}$$

$$(1-p)[\delta_j + \beta\overline{\varphi}/((1-p)\overline{\varphi} + \bar{\phi})]/(1+\delta_j) - \beta = 0. \tag{A.7}$$

Let $\Delta = \beta\overline{\varphi}/((1-p)\overline{\varphi} + \bar{\phi})$, then combining Eq. (A.6) and Eq. (A.7) leads to

$$(\delta_i/(1+\delta_i) - \delta_j/(1+\delta_j)) + \Delta(1/(1+\delta_i) - 1/(1+\delta_j)) = 0. \tag{A.8}$$

For Eq. (A.8) to be true, we need either $\Delta = 1$ or $\delta_i = \delta_j$. We can show that $\Delta = 1$ cannot be true. We prove this by contradiction. Assume that it is true, then we can substitute it into Eq. (A.6), and get $\beta = 1 - p$. Substituting $\beta = 1 - p$ into $\Delta = 1$, we get $\varphi = 0$. We know that $\varphi = 0$ is impossible given that $p \in (0, 1)$, thus $\Delta \neq 1$. Thus, the only possible solution is $\delta_i = \delta_j$, $\forall i, j$. This implies that $z_i^* = z_j^*$, i.e., $n_i^*/n_j^* = R_i/R_j$ at NE $\mathbf{n}_{ne}$ if it exists.

In the following, we prove that, when $n_i^*/n_j^* = R_i/R_j$, there exists a unique solution p^* for Eq. (A.2). Then we can conclude that there is one unique $\mathbf{n}_{ne}$.

Since at NE all players have the same number of paths, from Eq. (A.3), we obtain:

$$(m-1)/\beta - m/(1-p) + \overline{\varphi}/((1-p)\overline{\varphi} + \bar{\phi}) = 0. \tag{A.9}$$

Let $F(p)$ denote the left-hand-side of Eq. (A.9). Ideally, solving equation Eq. (A.9) with p as unknown, we can get the loss rate at NE p^*. Then, substituting p^* back into Eq. (A.9), we can get $\mathbf{n}_{ne}$ as the number of paths of all players at NE. However, Eq. (A.9) contains several powers of p such as 7/2 and 5/2, thus it is impossible to get an algebraic solution of p. Thus, in the following, we examine several properties of $F(p)$, from which we make an inference about the behavior of NE. For an exact value of p^* and $\mathbf{n}_{ne}$ when given a network setting, we can use Matlab to numerically solve for them.

First, we prove that Eq. (A.9) has only one solution for p in $(0, 1)$. Note that $F(p)$ is a continuous function, and the domain of $F(p)$ is $p \in (0, 1)$, and $\lim_{p\to 0} F(p) > 0$ and $\lim_{p\to 1} F(p) < 0$. We claim that $F(p)$ is a strictly monotonic decreasing function. If this claim is true, then there must be a single solution p^* for $F(p) = 0$. In the following, we prove this claim.

Consider the derivative

$$\frac{\partial F}{\partial p} = \frac{-m}{(1-p)^2} + \frac{\overline{\varphi}'\bar{\phi}}{[(1-p)\overline{\varphi} + \bar{\phi}]^2} < \frac{-1}{(1-p)^2} + \frac{\overline{\varphi}'\bar{\phi}}{[(1-p)\overline{\varphi} + \bar{\phi}]^2}$$

$$= \frac{-\bar{\phi}^2 - 2(1-p)\overline{\varphi}\bar{\phi} - (1-p)^2(\overline{\varphi}^2 - \overline{\varphi}'\bar{\phi})}{(1-p)^2[(1-p)\overline{\varphi} + \bar{\phi}]^2}. \tag{A.10}$$

Thus, to prove that $\partial F/\partial p < 0$, we only need to prove that $\overline{\varphi}^2 > \overline{\varphi}'\overline{\phi}$. This can be easily proved.

Then, substituting p^* back into Eq. (A.9), we can get n_i^* as the number of paths of the ith player at NE. Therefore, we conclude that there is only one NE for this game and it is symmetric. That is, $z_i^* = \arg\max_{n_i \in S_i} U_i(z_1^*, z_2^*, \ldots, z_i^*, \ldots, z_m^*)$ and $z_i^* = z_j^*$, thus $n_i^*/n_j^* = R_i/R_j$, $\forall i$ at NE $\mathbf{n}_{ne}$ if it exists.

Therefore, $F(p, m)$ is a monotonic decreasing function of p, and $F(p^*, m) = 0$. Since $F(p_0, m) < 0$, p^* must be smaller than p_0. When $p < p_0$, we get $dF/dm = 1/\beta - 1/(1-p) = 1/(1-p0) - 1/(1-p) > 0$. Thus, as m increases, $F(p, m)$ is strictly monotonic increasing. Since $F(p, m)$ is a monotonic decreasing function of p, we see that as m increases, for any given $F(p)$, p will be strictly increasing towards p_0. Then, it must be also true that for $F(p^*) = 0$, as m increases, p approaches to p_0.

Recall that at NE, we must have $(1 - p^*)/R_i\bar{\phi}^* = C/(mn_i^*)$. Since all players must have at least one path, i.e., $n_i^* \geq 1$, we must make sure that $m(1 - p^*)/R_i\bar{\phi}^* \leq C$. We know that as m increases, $p^* \to p_0 = 1 - \beta$, then $R_i\bar{\phi}^*$ (as a function of p^*) also increases to $R_i\bar{\phi}_0^*$ (function of p_0). Thus, $(1 - p^*)/R_i\bar{\phi}^*$ is bounded below by $(1 - p_0)/R_i\bar{\phi}_0^*$. So, as m becomes larger and larger, eventually, $m(1 - p^*)/R_i\bar{\phi}^*$ will be larger than C, and NE is no longer an interior point. Supposing R_1 is the minimum *RTT*, and $m(1 - p^*)/R_1\bar{\phi}^*$ is the largest value for all $m(1 - p^*)/R_i\bar{\phi}^*$, $\forall i$. Let m_0 denote this threshold, then it is the largest m satisfying $m(1-p^*)/R_1\bar{\phi}^* = m(1-p^*)B_1^* \leq C$. Since it is difficult to obtain an explicit expression of p as a function of m, we rely on the numerical method to identify m_0.

Appendix B

The appendix provides the proof of Theorem 4. Theorem 4 is restated as follows: There is a unique Nash Equilibrium (NE) $\mathbf{n}_{ne}$ in the continuous symmetric multipath selection game. At this NE, all players have the same number of paths. This NE is an interior point of the strategy space for $m < m_{0,\alpha}$ and $\mathbf{n}_{ne} = (1, 1, \ldots, 1)$ for $m \geq m_{0,\alpha}$, where $m_{0,\alpha}$ is given in Eq. (B.8).

Proof. Following a similar procedure in the proof of Theorem 1, we take derivative of utility function of player i with respect to its number of paths n_i:

$$\frac{\partial U_i}{\partial n_i} = \frac{C \sum_{k=1,k\neq i}^{m} n_k}{\left(n_i + \sum_{k=1,k\neq i}^{m} n_k\right)^2} - \frac{\beta}{\phi} - \frac{\beta n_i C \varphi}{\left(n_i + \sum_{k=1,k\neq i}^{m} n_k\right)^2 [(p-1)\varphi - \phi]} - \alpha\phi \tag{B.1}$$

$$\text{where}\quad \phi = 1/B = \mu R\sqrt{p} + T_0\nu\left(p^{3/2} + 32p^{7/2}\right) \tag{B.2}$$

$$\varphi = d\phi/dp = \frac{\mu R}{2\sqrt{p}} + T_0\nu\left(\frac{3}{2}\sqrt{p} + 112p^{5/2}\right). \tag{B.3}$$

Let $\delta_i n_i = \sum_{k=1,k\neq i}^{m} n_k$, then we get:

$$\frac{1-p}{1-\delta_i}\left[\delta_i + \frac{\beta\varphi}{(1-p)\varphi + \phi}\right] - \beta - \alpha\phi = 0. \tag{B.4}$$

Similarly, we can prove that $\delta_i = \delta_j$, $\forall i, j$, which means that at NE, all players have the same number of paths. Thus, Eq. (B.1) can be rewritten as:

$$\frac{m-1}{\beta} - \frac{m}{1-p} + \frac{\varphi}{(1-p)\varphi + \phi} - \frac{\alpha m\phi}{\beta(1-p)} = 0. \tag{B.5}$$

Let $F(p)$ denote the left-hand-side of this equation, we have:

(1) as $p \to 0$, $F(p) \to \frac{m-1}{\beta} - (m-1) > 0$.

(2) as $p \to 1$, $F(p) \to \frac{m-1}{\beta} - \infty + \frac{\varphi_1}{\phi_1} - \frac{\alpha}{\beta}\infty < 0$.

As in Theorem 1, we have $\frac{dF}{dp} < 0$. Thus, as $F(p)$ is a decreasing function, there must be unique p_a^* such that $F(p^*) = 0$. Solve Eq. (B.5) for p_a^* and substitute it into $\frac{\partial U_i}{\partial n_i}$, we can get a unique $\mathbf{n}_{ne} = (n_\alpha^*, n_\alpha^*, \ldots, n_\alpha^*)$, namely, the unique NE.

We also can show that if m gets very large, this unique NE will no longer be an interior point of the strategy space. Instead, it will be $\mathbf{n}_{ne} = \{1, 1, \ldots, 1\}$. Recall Eq. (B.5), and let

$$F(p, m) = m\left(\frac{1}{\beta} - \frac{1}{1-p} - \frac{\alpha\phi}{\beta(1-p)}\right) - \frac{1}{\beta} + \frac{\varphi}{(1-p)\varphi + \phi}. \tag{B.6}$$

Similarly as in the proof of Theorem 1, given a value of m, we can plot a curve for $F(p, m)$ with p as x-axis and $F(p, m)$ as y-axis. It can be easily verified that these curves (with different m values) all meet at a single common point $(p_0, F(p_0, m))$ with p_0 as a solution to $1 - p = \alpha\phi + \beta$.

Recall that $F(p, m)$ is a monotonic decreasing function of p, and $F(p_a^*,\ m) = 0$. Since $F(p_0,\ m) < 0$, p_{ne} must be smaller than p_0.

When $p < p_0$, we have:

$$\frac{dF}{dm} = \frac{1}{\beta} - \frac{1}{1-p} - \frac{\alpha\phi}{\beta(1-p)} > 0. \tag{B.7}$$

Combined with the result that $F(p, m)$ is a monotonic decreasing function of p, then p is an increasing function of m, and for $F(p_{ne}) = 0$, as m increases, p approaches to p_0.

Since in NE, we must have $B_\alpha^* \cdot (1 - p_{ne}) = C/(m \cdot n_{ne})$, and since all players must have at least one path, i.e., $\mathbf{n}_{ne} \geq 1$, we must make sure that $m \leq m_0$ where m_0 is the solution of

$$m(1 - p_\alpha^*)/\phi_\alpha^* = C. \tag{B.8}$$

Similarly as in the proof of Theorem 1, we know that as m increases, $p^* \to p_{0,\alpha}$, then ϕ_α^* (as a function of p_a^*) also increases to $\phi_{0,\alpha}$ (function of $p_{0,\alpha}$). Thus, $(1 - p_{ne})/\phi_\alpha^*$ is bounded. So, as m becomes larger and larger, eventually, $m(1 - p_{ne})/\phi_\alpha^*$ will be larger than C, which means that all players only use one path at NE. When this happens, the NE is no longer an interior point. After that, as m keeps increasing, p^* is larger than p_0 and eventually approaches to 1. Let $m_{0,\alpha}$ denote this threshold value, then we also need to rely on the numerical method to identify $m_{0,\alpha}$.

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