On the Connectivity of One-dimensional Vehicular Ad Hoc Networks

Liao Jianxin[1,2], Li Yuanzhen[1,2], Li Tonghong[3], Zhu Xiaomin[1,2], Zhang Lei[1,2]
[1]State Key Laboratory of Networking and Switching Technology, Beijing University of Posts and Telecommunications, Beijing 100876, China
[2]EBUPT Information Technology Co., Ltd, Beijing 100083, China
[3]Technical University of Madrid, Madrid Spain 28660

Abstract: In this paper we analyze connectivity of one-dimensional Vehicular Ad Hoc Networks where vehicle gap distribution can be approximated by an exponential distribution. The probabilities of Vehicular Ad Hoc Network connectivity for difference cases are derived. Furthermore we proof that the nodes in a sub-interval $[z_1, z_1 + \Delta z]$ of interval $[0, z], z > 0$ where all the nodes are independently uniform distributed is a Poisson process and the relationship of Vehicle Ad hoc Networks and one-dimensional Ad Hoc networks where nodes independently uniform distributed in $[z_1, z_1 + \Delta z]$ is explained. The analysis is validated by computing the probability of network connectivity and comparing it with the Mont Carlo simulation results.

Key words: Intelligent Transportation Systems (ITS); connectivity; Vehicular Ad Hoc Networks; one-dimensional Ad Hoc Networks

I. INTRODUCTION

Mobile Ad Hoc NETworks (MANETs) are comprised of self-organizing mobile nodes that lack a network infrastructure, such as base stations. This technology has been proposed as a complementary to the fourth-generation wireless networks[1]. And in recent years, there is a growing interest on the research and deployment of MANETs technology for vehicular communication, e.g. the PReVENT project in Europe, InternetITS in Japan, and Network on Wheels in Germany. In particular, these networks have important applications in intelligent transportation systems (ITS). The prospective applications of VANETs are categorized into two groups as comfort and safety applications. The first group is expected to improve the passengers' comfort and optimize traffic efficiency, whereas the second one improves driving safety.

For a one-dimensional Mobile Ad Hoc Networks (MANETs), assuming all the nodes are independently uniform distributed, a few approaches have been proposed in the literatures for computing the probability of connectivity[2-6]. Desai and Manjunath[2] derived a formula for the probability that the network G_1 (all the nodes are independently uniform distributed in a closed interval $[0, z], z > 0$) is connected. Gore[3, 4] gave some comments on[2] and derived another formula for the probability that G_1 is connected with a fixed node at point zero. The probability that the one dimensional Ad Hoc Networks is composed of at most C clusters is derived by Ghasemi and Nader-Esfahani in [5]. Reference [6] studies the connectivity of a 1-D ad hoc network with a user mobility model and derives a formula for the connectivity of a source destination pair.

For a special ad hoc networks, Vehicular Ad Hoc Networks(VANETs)[7], it is shown by Rudack et al.[8] that the vehicle gap distribution can be approximated by an exponential distribution with vehicle density $\mu = \frac{\lambda}{E[V]}$, where μ represents the number of vehicles per unit of distance, λ represents the mean arrival rate of vehicles, $E[V]$ represents the mean vehicular speed. The same conclusion is derived by S. Yousefi et al. in [9].

In this paper, we try to derive some connectivity properties of VANETs in highway scenarios. Firstly in Section II, we proof that the nodes number located in a sub-interval of $[0, z], z > 0$ approximately is a Poisson process and that under the condition that k vehicles locate in the interval $(0, L]$, the position of these vehicles, which are considered as unordered random variables, are independently and uniformly distributed in the interval $(0, L]$. These conclusions relate the previous works[2-6] to ours in this paper. Secondly in Section III, we get the following connectivity probability for VANETs in highway scenarios:

• The probability of connectivity of the i^{th} vehicle and the j^{th} vehicle ($j>i$).

• The probability of connectivity of the vehicles in the interval $[0, L](L>0)$.

• The probability of connectivity of a fixed node at point zero and the geographical point $L(L>0)$: An example for this scenario is Geocasting which means delivering a message from a source node to nodes in a given geographical region. In other words, the probability is the geographical point $L(L>0)$ is in multi hop transmission range of the fixed node at zero.

Nodes and vehicles are alternately used in this paper.

II. PRELIMINARIES

For a one-dimensional ad-hoc networks G_1, assuming n nodes are independently uniform distributed in a closed interval $[0, z](z > 0)$. The probability that there are k nodes in the interval $[z_1, z_1+\Delta z]$ ($z_1, \Delta z > 0$ and $z_1, z_1+\Delta z < z$) is given by the following theorem.

Theorem 1 The probability that there are k nodes in the interval $[z_1, z_1+\Delta z]$ ($z_1, \Delta z > 0$ and $z_1, z_1+\Delta z < z$) approximately is a Poisson point process when $\Delta z \ll z$ and $n \to \infty$.

$$P(X=k) \approx e^{-\mu}\frac{\mu^k}{k!} \tag{1}$$

where $p = \frac{\Delta z}{z}$, $\mu = np$.

Proof The probability that a node locates in the interval ($z_1, \Delta z > 0$ and $z_1, z_1+\Delta z < z$) is $p = P(z_1 < x_1 < z_1+\Delta z) = \frac{\Delta z}{z}$. So the probability that there are k nodes in the interval $[z_1, z_1+\Delta z]$ ($z_1, \Delta z > 0$) is:

$$P(X=k) = C_n^k p^k (1-p)^{n-k}$$

Let $np = \mu$, we get:

$$C_n^k p^k (1-p)^{n-k} = \frac{n!}{(n-k)!k!}\left(\frac{\mu}{n}\right)^k\left(1-\frac{\mu}{n}\right)^{n-k} = \frac{n(n-1)\dots(n-k+1)}{n^k}\frac{\mu^k}{k!}\frac{(1-\mu/n)^n}{(1-\mu/n)^k}$$

For n large and p small

$$(1-\frac{\mu}{n})^n \approx e^{-\mu}$$

$$\frac{n(n-1)\dots(n-k+1)}{n^k} \approx 1$$

$$(1-\frac{\mu}{n})^k \approx 1$$

Hence, for n large and p small,

$$P(X=k) \approx e^{-\mu}\frac{\mu^k}{k!} \ .$$

For Vehicular Ad Hoc Networks, we denote the Probability Density Function (PDF) and Cumulative Distribution Function (CDF) of the gap distribution as $f_g(x)$ and $F_g(x)$ respectively. Based on the conclusion from [8, 9], we have

$$f_g(x) = \mu e^{-\mu x} \tag{2}$$

$$F_g(x) = 1 - e^{-\mu x} \tag{3}$$

Obviously the number of nodes which locate in the interval $[z_1, z_1 + \Delta z]$ is a Poisson point process:

$$P(X=k) = \frac{(\mu\Delta z)^k}{k!}e^{-\mu\Delta z} \tag{4}$$

Theorem 2 Under the condition that k vehicles locate in the interval (0, z], the position of these vehicles, which are considered as unordered random variables, are independently and uniformly distributed in the interval (0, z].

Proof The conclusion is derived directly from Theorem 5[10].

Based on the discussion above, we get the conclusion: for one-dimensional Vehicular Ad Hoc Networks and one-dimensional Ad Hoc Networks where nodes uniformly distributed, the node number in an interval is a Poisson process and the nodes' position in an interval is uniformly distributed. Although our analysis scenario in the next section is Vehicular Ad Hoc Networks, our conclusions can also be applied in a one-dimensional Mobile Ad Hoc Networks where nodes uniform distributed.

III. PROBABILITY OF CONNECTIVITY

In this section, we try to calculate the probability of connectivity for one-dimensional Vehicular Ad Hoc Networks where vehicle gap distribution is an exponential distribution.

3.1 Connectivity probability of case 1

Theorem 3 The probability of connectivity of the i^{th} node and the j^{th} node($j > i$) is

$$P_{con}^{cs1} = F_g(R)^{j-i} \tag{5}$$

where R denotes the transmission range of radio.

Proof There are j i gaps between the i^{th} node and the j^{th} node. The probability of the distance of any two adjacent nodes less than R is $F_g(R)$. The conclusion is apparently derived.

3.2 Connectivity probability of case 2

Theorem 4 The probability of connectivity of the vehicles in the interval $[0, L](L > 0)$ is:

$$P_{con}^{cs2} = \sum_{k=0}^{\infty} P(X=k) \sum_{j=0}^{k-1}\binom{k-1}{j}(-1)^j \frac{(L-jR)^k}{L^k}u(L-jR) \tag{6}$$

where $u(x)$ is the unit step function.

Proof The probability of connectivity of the vehicles in the interval $[0, L](L > 0)$ is

$$P_{con}^{cs2} = \sum_{k=0}^{\infty} P(X=k) * p_c(k) \tag{7}$$

where $p_c(k)$ is the probability of connectivity of k nodes located in the interval $[0, L]$. The position of these vehicles are uniformly distributed in the interval (0, L] (see Theorem 2). This leads to $p_c(k) = p_c(k, L, R)$ where $p_c(k, L, R)$ can be derived by substituting k, L, R into the formula (8) in paper [2]. So we get:

$$p_c(k,L,R) = \sum_{j=0}^{k-1}\binom{k-1}{j}(-1)^j \frac{(L-jR)^k}{L^k}u(L-jR) \tag{8}$$

P_{con}^{cs2} is derived by substituting formula (8) into (7) and the proof is complete.

3.3 Connectivity probability of case 3

Theorem 5 The probability of connectivity of a fixed node at point zero and the geographical point $L(L > 0)$ is:

$$P_{con}^{cs3} = \sum_{k=\left\lceil \frac{L}{R} \right\rceil - 1}^{\infty} P(X=k) \sum_{j=0}^{k}\binom{k}{j}(-1)^j \frac{(L-jR)^k u(L-jR) - (L-(j+1)R)^k u(L-(j+1)R)}{L^k} \tag{9}$$

where $\lceil x \rceil$ is the least integer that is larger than x.

Proof Define $y_i = x_{i+1} - x_i$ as the distance of two adjacent nodes. Following the notation of [2-4] and using the subscript G for this case, let $U_G(n, L, R)$ $U_G(n, L, R)$ and $U_n(L)$ be the volumes of the polytopes $S_G(n, L, R)$ and $S(n, L)$.

$$\begin{aligned} S_G(n,L,R) := \{ & y_0, y_1, \ldots, y_{n-1} : y_i \geq 0 \text{ for } i \geq 0, \\ & y_i \leq R \text{ for } i > 0, \\ & L - R \leq \sum_{i=0}^{n-1} y_i \leq L \} \\ = \{ & y_0, y_1, \ldots, y_{n-1} : y_i \geq 0 \text{ for } i \geq 0, \\ & y_i \leq R \text{ for } i > 0, \sum_{i=0}^{n-1} y_i \leq L \} - \\ \{ & y_0, y_1, \ldots, y_{n-1} : y_i \geq 0 \text{ for } i \geq 0, \\ & y_i \leq R \text{ for } i > 0, \sum_{i=0}^{n-1} y_i \leq L - R \} \end{aligned} \tag{10}$$

$$S(n,L) := \{ y_0, y_1, \ldots, y_{n-1} : y_i \geq 0 \text{ for } i \geq 0, \sum_{i=0}^{n-1} y_i \leq L \} \tag{11}$$

$S_G(n, L, R)$ describes all feasible sequences of inter-node distances for which Networks in case 3 is connected. The probability that fixed node and the geographical point L are connected when there are n nodes in the networks is:

$$P_G(n,L,R) = \frac{U_G(n,L,R)}{U_n(L)} = \frac{U_G(n,L,R)}{\frac{L^n}{n!}} \tag{12}$$

Using the formula (*20*) in paper [3], we get:

$$U_G(n,L,R) = \sum_{j=0}^{n} \binom{n}{j} (-1)^j \frac{(L-jR)^n u(L-jR) - (L-(j+1)R)^n u(L-(j+1)R)}{L^n} \tag{13}$$

By substituting formula (13) into (12), $p_c(n, L, R)$ is derived. To ensure the connectivity of the fixed nodes and the point L, there are at least $\lceil \frac{L}{R} \rceil - 1$ nodes in the interval $[0, L]$.

$$P_{con}^{cs3} = \sum_{k=\lceil \frac{L}{R} \rceil - 1}^{\infty} P(X=k) P_G(k,L,R) \tag{14}$$

P_{con}^{cs3} is derived by substituting formula (12) into (14) and the proof is complete.

IV. DISCUSSION

In Figure 1 and Figure 2, P_{con}^{cs2} and P_{con}^{cs3} are plotted as a function of R/L for different values of $1/\mu$ respectively. Because the probability of connectivity of case 1 is simple and the analysis results tallies with the simulation perfectly, the results of case 1 are not showed.

Note that $1/\mu$ is the mean vehicle gap distance. In Figure 1, Figure 2, little $1/\mu$ leads to large connectivity probability. But in Figure 1 when $R/L<0.35$ which means L is larger than $R/0.35$, little $1/\mu$ leads to more nodes gap and any gap distance larger than R leads to disconnectivity. Also in Figure 1, as R/L increase, the connectivity probability increase until reaching their peak value then drop. The reason for the probability connectivity increase when R/L increase and before it reach their peak value is straightforward that large R/L means little L. The reason the curves decline after their peak value is little L means little node locate probability. In Figure 2, as R/L increase, L decrease. So

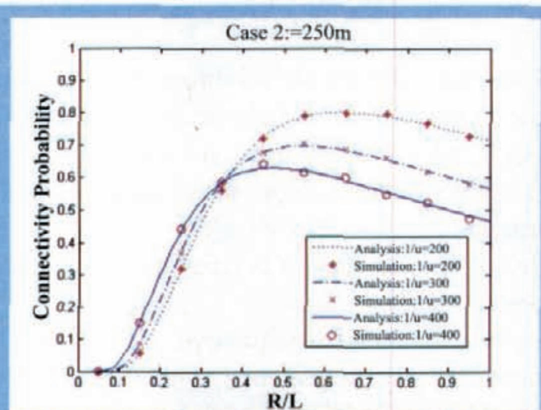

Fig.1 Probability of networks being connectivity for case 2

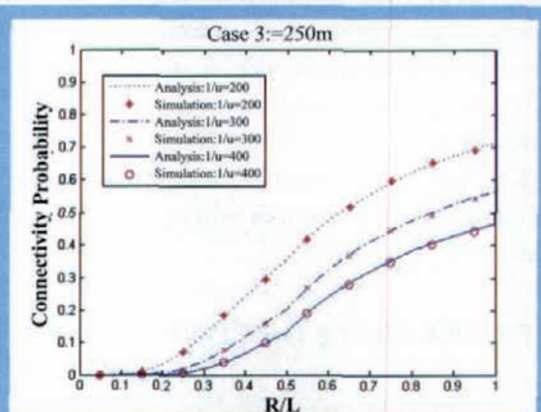

Fig. 2 Probability of networks being connectivity for case 3

connectivity probability is increase. When $R=L$, the connectivity probability is 1 because node at 0 can surely cover interval [0, R].

Since both the simulation and the analytical curves in Figure 1 and Figure 2 are close to each other in all the scenarios, we conclude that the analysis is fairly accurate. Although our simulation and analysis scenario is Vehicular Ad Hoc Networks, our conclusions can also be applied in a one-dimensional Mobile Ad Hoc Networks where nodes uniform distributed based Theorems in Section I. Our work can be used to networks planning to ensure the connectivity of the vehicle ad hoc networks. For $1/\mu=200$, $1/\mu=300$, $1/\mu=400$ respectively, when R/L is 0.64, 0.55, 0.47, the connectivity probability reach their peak value 0.801, 0.699, 0.629.

V. CONCLUSIONS

In this paper we analyze connectivity of one-dimensional Vehicular Ad Hoc Networks where vehicle gap distribution can be approximated by an exponential distribution and our conclusions can also be applied in a one-dimensional Mobile Ad Hoc Networks where nodes uniform distributed. We will use the probability derived in this paper in routing strategy for our future works. 中国通信

Acknowledgments

This paper was jointly supported by: (1) National Science Fund for Distinguished Young Scholars (No. 60525110); (2) National 973 Program (No. 2007CB307100, 2007CB307103); (3) National Natural Science Foundation of China (No. 60902051); (4) Chinese Universities Scientific Fund (BUPT2009RC0505); (5)Development Fund Project for Electronic and Information Industry (Mobile Service and Application System Based on 3G) .

References

[1] Frodigh, M., et al., Future-generation wireless networks. IEEE Perv. Comput. Mag., 2001. 8(5): p. 10-17.
[2] Desai, M. and D. Manjunath, On the Connectivity in Finite Ad Hoc Networks. IEEE Commun. Lett., 2002. 6(10): p. 437-439.
[3] Gore, A.D., Comments on "On the Connectivity in Finite Ad Hoc Networks". IEEE Commun. Lett., 2006. 10(2): p. 88-90.
[4] Gore, A.D., Correction to "Comments on 'On the Connectivity in Finite Ad Hoc Networks'". IEEE Commun. Lett., 2006. 10(5): p. 359.
[5] Ghasemi, A. and S. Nader-Esfahani, Exact probability of connectivity in one-dimensional ad hoc wireless networks. IEEE Commun. Lett., 2006. 10(4): p. 251- 253.
[6] Chuan, H.F., et al., Network connectivity of one-dimensional MANETs with random waypoint movement. IEEE Commun. Lett., 2005. 9(1): p. 31- 33.
[7] Yuanzhen, L., et al. A Benchmark for Delay Performance in Spare Vehicular Ad hoc Networks. in Wireless Communications, Networking and Mobile Computing, 2009. WiCOM '09. 5th International Conference on. 2009.
[8] Rudack, M., M. Meincke and M. Lott. On the Dynamics of Ad Hoc Networks for Inter Vehicle Communications (IVC). in International Conference on Wireless Networks ICWN'02, Jun 24-27, 2002. 2002. Las Vegas, Nevada, USA.
[9] Yousefi, S., et al., Analytical Model for Connectivity in Vehicular Ad Hoc Networks. IEEE Trans. Veh. Technol., 2008. 57(6): p. 3341-3356.
[10] Ross, S.M., Introduction to Probability Models. 2000, New York: Academic.

Biographies

Liao Jianxin was born in 1965, obtained his PhD degree at University of Electronics Science and Technology of China in 1996. He is presently a professor of Beijing University of Posts and Telecommunications. He has published hundreds of papers in different journals and conferences. His research interests are mobile intelligent network, broadband intelligent network and 3G core networks.

Li Yuanzhen was born in 1979, obtained his B.S. degree from Nanjing University of Science and Technology, 2004. He is currently working toward the Ph.D. degree at the State Key Laboratory of Networking and Switching Technology, Beijing University of Posts and Telecommunications. His research interests include communications software, Next Generation Network, and multimedia communication.

Li Tonghong was born in 1968, obtained his PhD degree from Beijing University of Posts and Telecommunications in 1999. He is currently an assistant professor with the department of computer science, Technical University of Madrid, Spain. His main research interests include resource management, distributed system, middleware, wireless networks, and sensor networks.

Zhu Xiaomin was born in 1974, obtained his PhD degree from Beijing University of Posts and Telecommunications in 2001. Now he is an associate professor in Beijing University of Posts and Telecommunications. His major is Telecommunications and Information Systems. His research interests span the area of intelligent networks and next-generation networks with a focus on 3G core network and protocol conversion. He has published over 110 papers, among which there are more than 30 first-authored ones, in different journals and conferences.

Zhang Lei was born in 1974, obtained his PhD degree from Beijing University of Posts and Telecommunications in 2004. He is currently a lecturer in Beijing University of Posts and Telecommunications. His main research interests include mobile intelligent network, 3G network and services.